OPERATORS WITH COMMON HYPERCYCLIC SUBSPACES

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Abstract. We provide a reasonable sufficient condition for a family of operators to have a common hypercyclic subspace. We also extend a result of the third author and A. Montes [22], thereby obtaining a common hypercyclic subspace for certain countable families of compact perturbations of operators of norm no larger than one.

1. Introduction

It is known that for any separable infinite dimensional Banach space X, there is a continuous linear operator $T : X \to X$ which is hypercyclic; that is, there is a vector x such that the set $\{x, Tx, \ldots, T^n x, \ldots\}$ is norm dense in X ([2], [5]). Moreover, a simple Baire category argument shows that the set $HC(T)$ of such so-called *hypercyclic vectors* x is a dense G_δ in X [21], and its linear structure is well understood: While $HC(T)$ must always contain a dense subspace ([9], [20]), it not always contains a *closed* infinite dimensional one; see [16] for a complete characterization of when this occurs. (Throughout, when we say that $HC(T)$ contains a vector space V we mean of course that every $x \in V$ *except* $x = 0$ is hypercyclic for T.) Thus, for example it was shown

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that for the simplest example of a hypercyclic operator on a Banach space, namely the Rolewicz operator

$$B_2 : \ell_2 \to \ell_2,\ B_2(x_1, x_2, \cdots) = 2(x_2, x_3, \cdots),$$

$HC(B_2)$ contains an infinite dimensional vector space but that this vector space cannot be closed [25, Theorem 3.4].

In recent years, an increasing amount of attention has been paid to the set $\cap_{T\in\mathcal{F}} HC(T)$ of common hypercyclic vectors of a given family $\mathcal{F}$ of hypercyclic operators acting on the same Banach space X. A trivial extension of the Baire argument alluded to above tells us that $\cap_{T\in\mathcal{F}} HC(T)$ is a dense subset of X whenever $\mathcal{F}$ is countable. Moreover, L. Bernal and C. Moreno [7] showed this set contains a dense vector space if we ask in addition that the members be hereditarily hypercyclic. Finally S. Grivaux proved that this additional hypothesis can be suppressed [17, Proposition 4.3].

Other important recent work is by E. Abakumov and J. Gordon [1], who showed that the uncountable intersection

$$\cap_{\{\lambda\in C\ \mid\ |\lambda|>1\}} HC(B_\lambda) \neq \emptyset,$$

where B_λ is the Rolewicz operator with 2 replaced by λ. In fact it is simple to derive from this that the intersection contains a dense subspace of ℓ_2. On the other hand, in [4] F. Bayart shows that under the assumption of a strong form of the hypercyclicity condition, uncountable collections of hypercyclic operators can indeed contain an infinite dimensional *closed* subspace of common hypercyclic vectors. Similar results were obtained by G. Costakis and M. Sambarino [13], who also provided a criterion for the existence of common hypercyclic vectors.

Our interest here will be in the following problem:

Problem 1. *Let $\mathcal{F}$ be a countable family of operators acting on a Banach space X. When does $\cap_{T\in\mathcal{F}} HC(T)$ contain a closed infinite dimensional subspace?*

After giving a natural example that indicates that the Abakumov-Gordon, Bayart and Costakis-Sambarino situations fail to hold in general for common subspaces, we prove the main result of this note, which extends a result by A. Montes [25, Theorem 2.1] by providing a reasonable sufficient condition on a countable family of hypercyclic operators acting on a Banach space to have a common infinite dimensional hypercyclic subspace (Corollary 3.5). We then apply this to extend a result of the third author and A. Montes [22], thereby obtaining a common hypercyclic subspace for certain countable families of operators of the form $T = U + K$ where $\|U\| \leq 1$ and K is compact.

2. EXAMPLE

Example 2.1. *Let $X = H$ be a separable, infinite-dimensional Hilbert space, and let S_H be the unit sphere of H. Let (w_n) be a sequence of positive scalars satisfying*

$$\lim_{n\to\infty} \inf_k \left(\prod_{j=1}^{n} w_{k+j}\right)^{\frac{1}{n}} \leq 1 \quad \text{and} \quad \limsup \prod_{j=1}^{n} w_j = \infty.$$

For each h in S_H, let $\{e(h)_n : n \geq 1\}$ be a basis of H with $e(h)_1 = h$, and let $T_h : H \to H$ be the corresponding unilateral weighted backward shift defined by

$$T_h e(h)_n = \begin{cases} 0 & \text{if } n = 1 \\ w_n\, e(h)_{n-1} & \text{if } n \geq 2, \end{cases} \tag{1}$$

So T_h *has a hypercyclic subspace* [23, Corollary 2.3]*. Also, notice that* $\mathcal{F} = \{B_h : \ h \in S_H\}$ *satisfies that for all* $0 \neq y$ *in* H*,*

$$T_{\frac{y}{\|y\|}} y = 0.$$

That is, $\mathcal{F}$ *is a family of operators, each one having a hypercyclic subspace, but such that there is no hypercyclic vector common to all members of* $\mathcal{F}$*.*

Let us also observe that in [1] the authors mention that there is no common hypercyclic vector for the family of hypercyclic operators $\{\lambda B \oplus \delta B \ : \ |\lambda|, |\delta| > 1\}$. It is easy to see that no operator in this family admits a hypercyclic subspace.

3. A sufficient condition for a common hypercyclic subspace

We prove the main result in the more general setting of universality. Given a sequence $\mathcal{F} = \{T_j\}_{j\in\mathbb{N}}$ of bounded operators acting on a Banach space X, we say that a vector $x \in X$ is *universal* for $\mathcal{F}$ if $\{Tx : \ T \in \mathcal{F}\}$ is dense in X; the set of such universal vectors is denoted $HC(\mathcal{F})$. The sequence $\mathcal{F}$ is said to be *universal* (respectively, *densely universal*) provided $HC(\mathcal{F})$ is non-empty (respectively, dense in X). $\mathcal{F}$ is called *hereditarily universal* (respectively, *hereditarily densely universal*) provided $\{T_{n_k}\}_{k\in\mathbb{N}}$ is universal (respectively, densely universal) for each increasing sequence (n_k) of positive integers. For more on the notion of universality, see [15] and [19]. A result similar to the following theorem is proved in [10] for a (unique) sequence of universal operators in the context of Fréchet spaces.

Theorem 3.1. *Let $T_{n,j}$ $(n, j \in \mathbb{N})$ be bounded operators on a Banach space X, and let Y be a closed subspace of X of infinite dimension. Suppose that for each $n \in \mathbb{N}$*

i) *$\{T_{n,j}\}_{j\in\mathbb{N}}$ is hereditarily densely universal, and*
ii) *$\lim_{j\to\infty} \|T_{n,j}x\| = 0$ for each x in Y.*

Then there exists a closed, infinite dimensional subspace X_1 of X so that $\{T_{n,j}x\}_{j\in\mathbb{N}}$ is dense in X for each non-zero $x \in X_1$ and $n \in \mathbb{N}$. In particular, X_1 is a universal subspace of $\{T_{n,j}\}_{j\in\mathbb{N}}$ for each $n \in \mathbb{N}$.

Lemma 3.2. *Let $T_{n,j}$ $(n, j \in \mathbb{N})$ be bounded operators on a Banach space X so that for each fixed integer n the family $\{T_{n,j}\}_{j\geq 1}$ is densely universal. Then the set $\cap_{n=1}^{\infty} HC(\{T_{n,j}\}_{j\geq 1})$ of common universal vectors to every sequence $\{T_{n,j}\}_{j\in\mathbb{N}}$ is dense in X.*

Proof. $\cap_{n=1}^{\infty} HC(\{T_{n,j}\}_{j\geq 1})$ is a countable intersection of dense G_δ subsets of the Baire space X [18, Satz 1.2.2]. □

Proof of Theorem 3.1. Reducing the subspace Y if necessary, we may assume it has a normalized Schauder basis $(e_j)_j$. Let (e_j^*) be its associated sequence in Y^* of coordinate functionals, that is, so that $e_j^*(e_i) = \delta_{i,j}$ for $i, j \in \mathbb{N}$. Let $A(Y, X)$ denote the norm closure of the subspace

$$\left\{ \sum_{j=1}^{n} x_j e_j^*(\cdot) : \ n \in \mathbb{N}, \ x_1, \ldots, x_n \in X \right\}.$$

For each T in $B(X)$, define $L_T : A(Y, X) \to A(Y, X)$ by $L_T V := TV$. We make use of the following lemma, whose proof follows that of Theorem 3.1. Analogous versions of this lemma are proved in [10] for several operator ideals (nuclear, compact, approximable), in a more general context, by using tensor product techniques developed in [24].

Lemma 3.3. *Suppose $\{T_j\}_{j\in\mathbb{N}}$ is a sequence of bounded operators on X that is hereditarily densely universal. Then $\{L_{T_{r_j}}\}_{j\geq 1}$ is a hereditarily densely universal sequence of operators on $A(Y,X)$, for some increasing sequence (r_j) of positive integers.*

Now, notice that by (i) and Lemma 3.3, for each fixed $n\in\mathbb{N}$ there exists a sequence of positive integers $(r_{n,j})_j$ so that the sequence of operators $\{L_{T_{n,r_{n,j}}}\}_{j\in\mathbb{N}}$ is hereditarily densely universal on the Banach space $A(Y,X)$. By Lemma 3.2, there exists V in $A(Y,X)$ that is universal for every sequence $\{L_{T_{n,r_{n,j}}}\}_{j\in\mathbb{N}}$, and hence universal for every $\{L_{T_{n,j}}\}_{j\in\mathbb{N}}$, too ($n\in\mathbb{N}$). Multiplying V by a non-zero scalar if necessary, we may assume that $\|V\|<\frac{1}{2}$. Consider now $X_1:=(i+V)(Y)$, where $i:Y\to X$ is the inclusion. For each $x\in Y$, $\|(i+V)x\|\geq\|x\|-\|Vx\|\geq\frac{1}{2}\|x\|$. So $i+V$ is bounded below and X_1 is closed and of infinite dimension. Notice that $\{T_{n,j}Vx\}_{j\in\mathbb{N}}$ is dense in X for every $0\neq x\in Y$ and every $n\in\mathbb{N}$. Indeed, given $\epsilon>0$, let $z\in X$ be arbitrary, and let S be a finite rank operator in $A(Y,X)$ such that $Sx=z$. By Lemma 3.3, for each n there is some $T_{n,j}$ such that $||T_{n,j}V-S||<\frac{\epsilon}{\|x\|}$. In particular, $||T_{n,j}Vx-Sx||=||T_{n,j}Vx-z||<\epsilon$. The theorem now follows from condition (ii). □

Proof of Lemma 3.3. Since $\{T_j\}_{j\in\mathbb{N}}$ is hereditarily densely universal on X, it follows from [6, Theorem 2.2] that there exists a dense subspace X_0 of X, an increasing sequence of positive integers (r_j) and (possibly discontinuous) linear mappings $S_j:X_0\to X$ ($j\in\mathbb{N}$) so that

$$T_{r_j},S_j,\ \text{and}\ (T_{r_j}S_j-I)\underset{j\to\infty}{\to}0 \tag{2}$$

pointwise on X_0. Now, consider

$$A_0:=\{V\in A(Y,X):V(Y)\subset X_0\ \text{ and }\dim(V(Y))<\infty\}.$$

Then A_0 is dense in $A(Y,X)$, and it follows from (2) that

$$L_{T_{r_j}}, L_{S_j}, \text{ and } [L_{T_{r_j}} L_{S_j} - I] \underset{j\to\infty}{\longrightarrow} 0$$

pointwise on A_0. So $\{L_{T_{r_j}}\}_{j\geq 1}$ is hereditarily densely universal on $A(Y,X)$, by [6, Theorem 2.2]. □

Remark 3.4. *An alternative constructive proof of Theorem 3.1 may be done with the arguments from* [25, Theorem 2.2]. *The proof here is much simpler, and follows arguments from* [10] *and* [11].

Corollary 3.5. *Let T_l ($l \in \mathbb{N}$) be operators acting on a Banach space X. Suppose there exists a closed, infinite dimensional subspace Y of X, increasing sequences $(n_{l,q})_q$ of positive integers, and scalars $c_{l,q}$ so that for $l \in \mathbb{N}$*

i) $\{c_{l,q} T_l^{n(l,q)}\}_{q\in\mathbb{N}}$ *is hereditarily universal, and*
ii) $\lim_{q\to\infty} \|c_{l,q} T_l^{n(l,q)} x\| = 0$ *for each x in Y.*

Then there exists a closed, infinite dimensional subspace X_1 of X so that $\{c_{l,q} T_l^{n(l,q)} x\}_{q\in\mathbb{N}}$ is dense in X for each non-zero $x \in X_1$ and each $l \in \mathbb{N}$. That is, X_1 is a supercyclic subspace for T_l for every $l \in \mathbb{N}$. Moreover X_1 is a hypercyclic subspace for T_l for every $l \in \mathbb{N}$ if the constants $c_{l,q}$ are of modulus one.

4. An Application to Countable Families of Operators

We now apply Theorem 3.1 to show the following extension of [22, Theorem 4.1] to countable families of operators.

Theorem 4.1. *Let $\mathcal{F} = \{T_l = U_l + K_l : l \in \mathbb{N}\}$ be a family of operators acting on a common Banach space X. Suppose that for each $l \in \mathbb{N}$*

a) $\|U_l\| \leq 1$, *K_l is compact, and*

b) $\{T_l^{n_{l,q}}\}_{q\geq 1}$ *is hereditarily universal, for some increasing sequence* $(n_{l,q})_{q\geq 1}$ *of positive integers.*

Then the operators in $\mathcal{F}$ *have a common hypercyclic subspace.*

To show Theorem 4.1, we make use of the following lemmas. The first one follows from a slight modification of a proof by Mazur [14, p 38-39]. The second one is [16, Lemma 2.3] The last one is proved at the end of this section.

Lemma 4.2. *Let* (X_n) *be a sequence of closed, finite-codimensional subspaces of* X*, with* $X_n \supseteq X_{n+1}$ $(n \geq 1)$*. Then there exists a normalized basic sequence* (e_n) *so that* e_n *belongs to* X_n *for all* $n \geq 1$*.*

Lemma 4.3. [16, Lemma 2.3] *Let* $\{T_l^{n_{l,q}}\}_q$ *be hereditarily hypercyclic* $(l \in \mathbb{N})$*. Then there exists a dense subset* X_0 *of* X *and, for each* $l \in \mathbb{N}$*, a subsequence* $(r_{l,q})_q$ *of* $(n_{l,q})_q$ *so that*

$$\lim_{q\to\infty} \|T_l^{r_{l,q}}x\| = 0 \qquad (x \in X_0).$$

Lemma 4.4. *Let* X *and* Z *be Banach spaces, and let* $K_{l,n} : X \to Z$ *be compact operators* $(l, n \geq 1)$*. Given* $\epsilon > 0$*, there exist closed linear subspaces* X_n *of finite codimension in* X $(n \geq 1)$ *so that*

i) $X_n \supseteq X_{n+1}$

ii) $\|K_{l,n}x\| \leq \epsilon \|x\| \quad (x \in X_n,\ 1 \leq l \leq n)$

Proof of Theorem 4.1. Because $\{T_l^{n_{l,q}}\}_{q\geq 1}$ is hereditarily densely universal for each $l \in \mathbb{N}$, by Theorem 3.1 it suffices to get a closed, infinite dimensional subspace Y of X and subsequences $(m_{l,q})_q$ of $(n_{l,q})_q$ so that

$$\lim_{q\to\infty} \|T_l^{m_{l,q}}x\| = 0 \qquad (x \in Y,\ l \in \mathbb{N}).$$

For each pair of positive integers n and l, let $K_{l,n}$ be the compact operators defined by $T_l^n = (U_l + K_l)^n = U_l^n + K_{l,n}$. Apply Lemma 4.4 to get closed, finite codimensional subspaces X_n of X satisfying

$$\text{(3)} \qquad \begin{cases} a) \quad X_n \supseteq X_{n+1} \\ b) \quad \|K_{l,n}x\| \leq \|x\| \quad (x \in X_n, 1 \leq l \leq n). \end{cases}$$

By Lemma 4.2, we can pick a normalized basic sequence (e_n) in X so that $e_n \in X_n$ $(n \in \mathbb{N})$. Let $K > 0$ be the basis constant of (e_n), and pick a decreasing sequence of positive scalars, (ϵ_m), so that $\sum_{n=1}^{\infty} \epsilon_n < \frac{1}{2K}$. By Lemma 4.3, there exist subsequences $(\tilde{n}_{l,q})_q$ of $(n_{l,q})_q$ and a dense subspace X_0 of X so that

$$\text{(4)} \qquad \lim_{q \to \infty} \|T_l^{\tilde{n}_{l,q}} x\| = 0 \qquad (x \in X_0).$$

Pick a sequence (z_m) in X_0 so that

$$\text{(5)} \qquad \|e_n - z_n\| < \frac{\epsilon_n}{\max\{\, \|T_l^i\| : \ l, i \leq n. \,\}}.$$

Notice that $\|e_n - z_n\| < \epsilon_n$ $(n \geq 1)$ and, because (e_n) is normalized, $|e_n^*(x)| \leq 2K\|x\|$ $(n \geq 1)$ for all x in $Y_0 = \overline{\text{span}\{e_1, e_2, \dots\}}$, where (e_n^*) is the sequence of functional coefficients associated with the Schauder basis (e_n) of Y_0. Hence $\sum_{n=1}^{\infty} \|e_n^*\| \, \|e_n - z_n\| < 2K \sum_{n=1}^{\infty} \epsilon_n < 1$, and so any subsequence (z_{n_k}) of (z_m) is equivalent to the corresponding basic sequence (e_{n_k}) [14, p 46]. We let $Y := \overline{\text{span}\{z_{n_k} : k \geq 1\}}$, where $(z_{n_k}) \subseteq (z_n)$ is defined as follows. Let $n_0 := 1$. For $l \in \mathbb{N}$, choose $m_{l,1}$ in $(\tilde{n}_{l,q})$ so that $\|T_l^{m_{l,1}} z_{n_0}\| < \frac{\epsilon_{n_0}}{2}$. Also, let $n_1 := m_{1,1}$. Next, for each $l \in \mathbb{N}$, since $z_{n_0}, \ z_{n_1} \in X_0$, we may apply (4) to get $m_{l,2} \in (\tilde{n}_{l,q})_q$ which satisfies the following conditions.

$$\begin{cases} \qquad m_{l,2} & > \max\{2, n_1, m_{l,1}\} \\ \|T_l^{m_{l,2}} z_{n_i}\| & < \frac{\epsilon_{n_i}}{2^2} \qquad i = 0, 1. \end{cases}$$

Also, let $n_2 := \max_{1\le l\le 2}\{m_{l,2}\}$. Continuing this process we get, for each $l \in \mathbb{N}$, an integer $m_{l,s}$ in $(\tilde{n}_{l,q})_q$ so that

$$\begin{cases} i) \quad m_{l,s} > \max\{s, n_{s-1}, m_{l,s-1}\} \\ ii) \ \|T_l^{m_{l,s}} z_{n_i}\| < \frac{\epsilon_{n_i}}{2^s} \qquad i = 0, \dots, s-1, \end{cases} \tag{6}$$

where $n_r = \max_{1\le l\le r}\{m_{l,r}\}$ for each $r \in \mathbb{N}$. It suffices to show that $T_l^{m_{l,s}} \underset{s\to\infty}{\to} 0$ pointwise on Y ($l \in \mathbb{N}$). Let $0 \neq z = \sum_{j=1}^{\infty} \alpha_j z_{n_j}$ in Y, $l \in \mathbb{N}$ be fixed, and $s \geq l$ be arbitrary. Then

$$T_l^{m_{l,s}} z = \sum_{j=1}^{s-1} \alpha_j T_l^{m_{l,s}} z_{n_j} + \sum_{j=s}^{\infty} \alpha_j \, T_l^{m_{l,s}}(z_{n_j} - e_{n_j}) + T_l^{m_{l,s}}(\sum_{j=s}^{\infty} \alpha_j \, e_{n_j}). \tag{7}$$

Notice that $|\alpha_j| \leq 2L\|z\| \quad (1 \leq j)$, where L is the basis constant of (z_{n_k}). By (6.ii),

$$\|\sum_{j=1}^{s-1} \alpha_j T_l^{m_{l,s}} z_{n_j}\| < \sum_{j=1}^{s-1} |\alpha_j| \frac{\epsilon_{n_j}}{2^s} \leq \frac{L\|z\|}{2^{s-1}} \sum_{j=1}^{s-1} \epsilon_{n_j}. \tag{8}$$

Also, by (6i) and (5)

$$\|\sum_{j=s}^{\infty} \alpha_j \, T_l^{m_{l,s}}(z_{n_j} - e_{n_j})\| \leq \ 2L\|z\| \sum_{j=s}^{\infty} \epsilon_{n_j}. \tag{9}$$

Finally, since $X_{n_s} \subseteq X_{m_{l,s}}$ and $\|U_l\| \leq 1$, by (3b)

$$\begin{aligned} \|T_l^{m_{l,s}} \sum_{j=s}^{\infty} \alpha_j \, e_{n_j}\| &= \|(U_l^{m_{l,s}} + K_{l,m_{l,s}})(\sum_{j=s}^{\infty} \alpha_j \, e_{n_j}) \ \| \\ &\leq \ 2 \ \|\sum_{j=s}^{\infty} \alpha_j \, e_{n_j}\| \qquad (s \geq l). \end{aligned} \tag{10}$$

So by (7), (8), (9), and (10), $\lim_{s\to\infty} \|T_l^{m_{l,s}} z\| = 0$. We finish the proof of Theorem 4.1 by showing Lemma 4.4.

Proof of Lemma 4.4. Let $n \geq 1$ and $\epsilon > 0$ be fixed. Because each ${K_{l,n}}^* : Z^* \to X^*$ is compact, there exist $z^*_{l,n,1}, \dots, z^*_{l,n,k_{l,n}}$ in X^* so that

$$K^*_{l,n}(B_{Z^*}) \subseteq \cup_{i=1}^{k_{l,n}} \ B(z^*_{l,n,i}, \epsilon). \tag{11}$$

For each positive integer s, let $X_s := \cap_{n=1}^{s} \cap_{l=1}^{n} \cap_{i=1}^{k_{l,n}} \mathrm{Ker}(z^*_{l,n,i})$. So each X_s is closed and of finite codimension in X, and $X_s \supseteq X_{s+1}$ $(s \geq 1)$. Now, let $x \in X_n$, and let $1 \leq l \leq n$ be fixed. By the Hahn-Banach theorem, there is a functional x^* of norm one so that $\|K_{l,n}x\| = \langle K_{l,n}x, x^*\rangle$. By (11), we may choose $1 \leq j \leq k_{l,n}$ so that $\| K^*_{l,n}x^* - z^*_{l,n,j}\| < \epsilon$. Hence, because x is in $X_n \subseteq \mathrm{Ker}(z^*_{l,n,j})$, $\|K_{l,n}x\| = \langle x, K^*_{l,n}x^* - z^*_{l,n,j}\rangle \leq \epsilon \,\|x\|$. □

The proof of Theorem 4.1 is now complete. □

We'd like to finish with the following two problems.

Problem 2. *Let T_1, T_2 be two operators acting on a Banach space X. Suppose each of T_1, T_2 has a hypercyclic subspace (i.e., a closed, infinite dimensional subspace of hypercyclic vectors). Must they share a common hypercyclic susbpace?*

Problem 3. *Let T_1, T_2 be two hereditarily hypercyclic operators acting on a Banach space X, with a common hypercyclic subspace. Must there exist sequences $(n_{l,q})_q$ $(l = 1, 2)$ and a closed infinite dimensional subspace Y of X so that $\{T_l^{n_{l,q}}\}_q$ is hereditarily hypercyclic and $T_l^{n_{l,q}} \underset{q\to\infty}{\longrightarrow} 0$ pointwise on Y $(l = 1, 2)$?*

References

[1] E. Abakumov and J. Gordon, *Common hypercyclic vectors for multiples of the backward shift*, J. Funct. Anal. 200 (2003), 494504.

[2] S. I. Ansari, *Existence of hypercyclic operators on topological vector spaces,* J. Funct. Anal. **148** (1997), 384-390.

[3] S. I. Ansari, *Hypercyclic and cyclic vectors*, J. Funct. Anal. **128** (1995), 374-383.

[4] F. Bayart, *Common hypercyclic subspaces,* preprint.

[5] L. Bernal-González, *On hypercyclic operators on Banach space,* Proc. Amer. Math. Soc. **127** (1999), 1003-1110.

[6] L. Bernal-González and K.-G. Grosse Erdmann, *The Hypercyclicity Criterion for sequences of operators,* Studia Mathematica, **157** (2003), 17-32.

[7] L. Bernal-González and M. C. Calderón-Moreno, *Dense linear manifolds of monsters,* J. Approx. Theory 119 (2002), 156–180.

[8] J. Bès and A. Peris, *Hereditarily Hypercyclic Operators,* J. Funct. Anal., **167** (1999), 94-112.

[9] P. Bourdon, *Invariant Manifolds of Hypercyclic Vectors,* Proc. Amer. Math. Soc. **118** No. 3 (1993), 845-847.

[10] J. Bonet, F. Martínez-Giménez, and A. Peris, *Universal and chaotic multipliers on operator ideals,* preprint.

[11] K. C. Chan and R. D. Taylor, *Hypercyclic subspaces of a Banach space,* Integr. equ. oper. theory, **41** (2001) 381-388.

[12] J. Conway, A course in Functional Analysis, 2nd edition, Springer-Verlag, New York, 1990.

[13] G. Costakis & M. Sambarino, *Genericity of wild holomorphic functions and common hypercyclic vectors,* Adv. Math., to appear.

[14] J. Diestel, *Sequences and Series in Banach Spaces,* Springer-Verlag, New York, 1984

[15] G. Godefroy and J. H. Shapiro, *Operators with dense, invariant cyclic vector manifolds,* J. Funct. Anal. **98**, 229-269 (1991).

[16] M. González, F. León-Saavedra, and A. Montes-Rodríguez, *Semi-Fredholm Theory: hypercyclic and supercyclic subspaces,* Proc. London Math. Soc.(3) **81** (2000), n$^{\underline{\circ}}$1, 169-189.

[17] S. Grivaux, *Construction of operators with prescribed behaviour,* Arch. Math., to appear.

[18] K.-G. Grosse-Erdmann, *Holomorphe Monster und universelle funktionen*, Mitt. Math. Sem. Giessen **176** (1987).

[19] K.-G. Grosse-Erdmann, *Universal Families and Hypercyclic Operators,* Bull. Amer. Math. Soc. **36** (1999), 345-381.

[20] D. Herrero, *Limits of hypercyclic and supercyclic operators,* J. Funct. Anal. **99** (1991) 179-190.

[21] C. Kitai, *Invariant Closed Sets for Linear Operators,* Ph. D. Thesis, Univ. of Toronto, 1982.

[22] F. León-Saavedra & A. Montes-Rodríguez, *Linear structure of hypercyclic vectors,* J. Funct. Anal. **148** (1997), 524-545.

[23] F. León-Saavedra & A. Montes-Rodríguez, *Spectral Theory and Hypercyclic Subspaces* Trans. Amer. Math. Soc. **353** (2001), n°1, 247-267.

[24] F. Martínez-Giménez & A. Peris, *Universality and chaos for tensor products of operators,* J. Approx. Theory, to appear.

[25] A. Montes-Rodríguez, *Banach spaces of hypercyclic vectors,* Michigan Math. J. **43** (1996), 419-436.

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