| 15 | 20 | 15 | 10 | 52 | 112 more. |
|---|---|---|---|---|---|
| A | B | C | D | M | Total |

NAME.....

Intuitive Calculus 11012 Examination 2 **A**
March 11, 2010 Richard M. Aron

**Directions: Please answer questions A, B, C, and D in the space provided.** ***Please write extremely neatly.*** **The rest of the Examination questions are to be answered on the "scan-tron" papers,** ***but you must show your work on this paper—even for the "scan-tron" part. Good luck!***

Product Rule: if $p(x) = f(x) \cdot g(x)$, then $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.
Quotient Rule: if $q(x) = \frac{f(x)}{g(x)}$, then $q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$.

A. Let $f(x) = x^4 - 4x^3 + 4x^2$ on the interval $[0, 3]$.
(a). Find all critical numbers of $f(x)$.

$4x^3 - 12x^2 + 8x = 4x(x^2 - 3x + 2) = 0 \Leftrightarrow x = 0, 1, 2.$

(b). Classify the critical numbers found in part (a). (That is, explain whether a particular critical number corresponds to a relative maximum or a relative minimum of $f(x)$.)

$y' = 12x^2 - 24x + 8 = 4(3x^2 - 6x + 2)$.

$y''(0) = 8 > 0$ min
$y''(1) = -4 < 0$ max
$y''(2) = 8 > 0$ min.

(c). Find the absolute extreme values of $f(x)$ on the closed interval $[0, 3]$.

$y(0) = 0$
$y(1) = 4$
$y(2) = 0$
$y(3) = 81 - 108 + 36 = 9$

0, 9

B. A car dealer can sell four cars per day at a price of \$12,000. She estimates that for each \$200 price reduction, she can sell two more cars per day. What price should she charge to maximize her revenue R(x)? How many cars will she sell each day? [*Hint*: Let $x$ = the number of \$200 price reductions.]

Step 1. Let $p(x)$ be the price she charges for a car, after $x$ price reductions.

$p(x) = 12000 - 200x$

Step 2. Let $q(x)$ be the quantity of cars she sells in a day, after $x$ price reductions.

$q(x) = 4 + 2x$

Step 3. Calculate $R(x)$, ~~and the number of cars that are sold~~.

$R(x) = (12000 - 200x)(4 + 2x)$

$= 48000 + 23200x - 400x^2$

Step 4. Maximize $R(x)$, explaining why your answer gives a maximum.

$R'(x) = 23200 - 800x$ , $R'' = -800 < 0$, max.

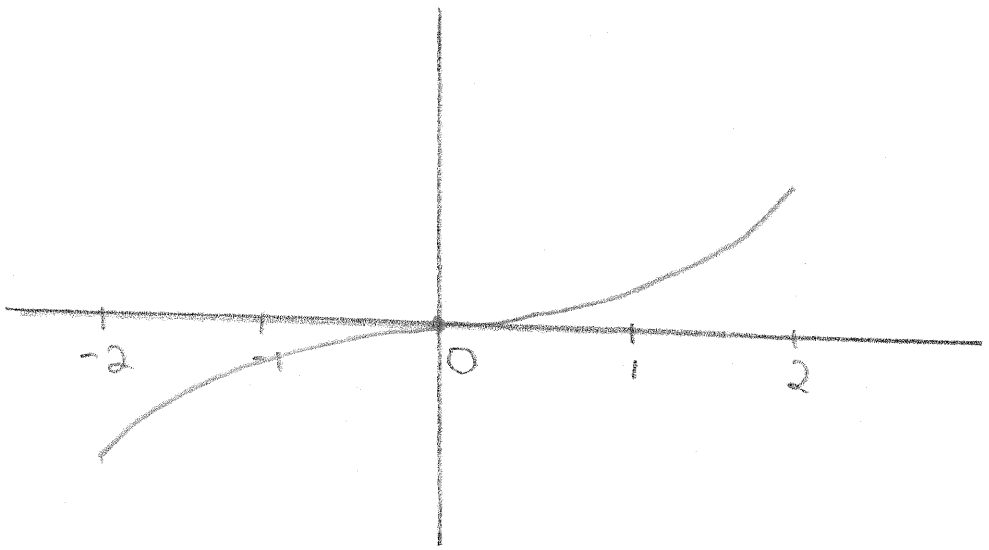
$x = \frac{232}{8} = 29$

Step 5. What price should she charge to maximize her revenue? How many cars will she sell each day?

$p(x) = 12000 - 200(29) = 6200$

$q(x) = 4 + 29 \cdot 2 = 62$

C. On the axes below, draw an *extremely clear, careful* sketch of the graph of a function $f$ which has the following properties: $f$ is continuous and differentiable, $f(0) = 0, f'(x) > 0$ for $x \in (0, 2)$, $f$ is concave down on $(-2, 0)$ and $f''(x) > 0$ on $(0, 2)$. Indicate any relative maxima or minima, and any inflection points.




D.(a). Evaluate. $f'$ and $f''$ for $f(x) = 12\sqrt{x^3} - 9\sqrt[3]{x}$.

(a). $f'(x) =:$ $y = 12x^{3/2} - 9x^{1/3}$

$y' = 18x^{1/2} - 3x^{-2/3}$

(b). $f''(x) =:$

$y'' = 9x^{-1/2} + 2x^{-5/3}$

**The rest of this Examination is to be done using the "scantron" sheet. Please write your work on this paper.**

1. Let $f(x) = (2x+1)^{10}$. Then $f'(x) =$:

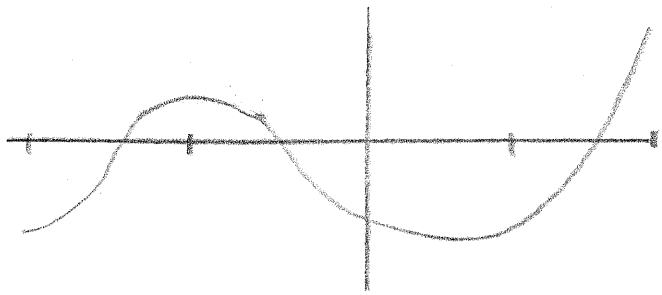
(a). $10(2x+1)^9$.

(b). $20(2x+1)^9$.

(c). $10(2x)^9$.

(d). None of the above answers.

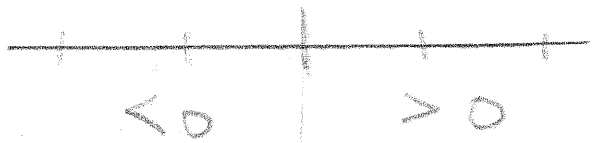
2. Let $y = f(x)$ be a differentiable function on $[-2, 2]$, whose graph is given below.



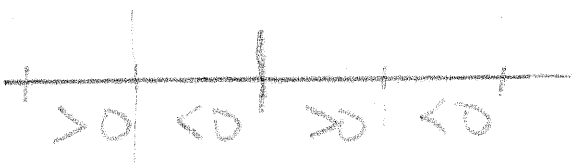
Which of the following is the sign diagram for $f'$?

(a)

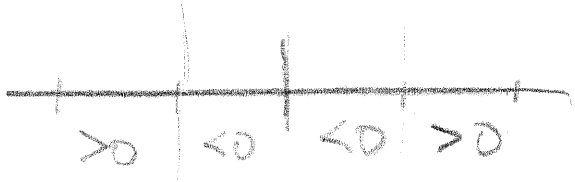

(b)




(c)



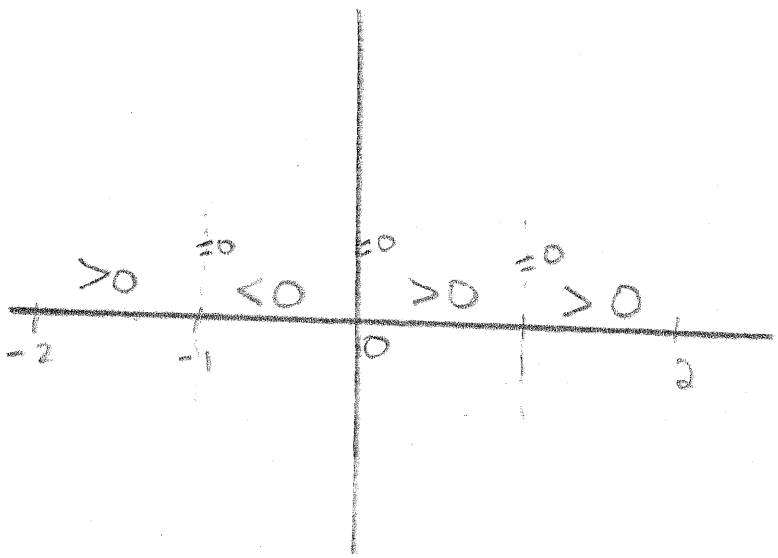

(d)



3. Let $y = f(x) = 2x^3 - 3x^2 - 12x$, where $x$ is in the closed interval $[-2, 4]$. Then the smallest and largest values that the function attains in this interval are:
(a). 0 and $\infty$.
(b). $-20$ and 7.
(c). $-\infty$ and $\infty$.
(d). $-20$ and 32.

4. The point $x = 0$ is where the function $y = f(x) = x^4 + 6$ has:
(a). a relative maximum.
(b). a relative minimum.
(c). neither a relative maximum nor a relative minimum.
(d). Answer cannot be determined from the given information.

5. Let $y = f(x)$ be a function such that $f'(x)$ has the following sign diagram.

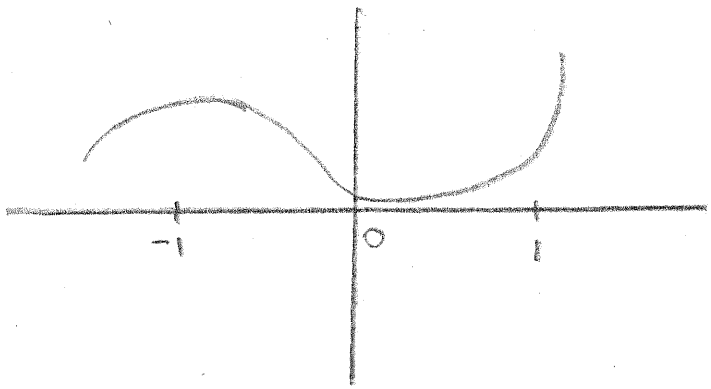



Which of the following is true? $f$ has:
(a). a relative maximum at $-1$.
(b). a relative maximum at $-1$ and a relative minimum at 0.
(c). relative maxima at $-1$ and at 1.
(d). a relative minimum at $-1$ and a relative maximum at 1.

6. The derivative of $y = f(x) = (x^{10} + 3x^5 - 2x^3 - 4)^6$ is:
(a). $10(x^9 + 15x^4 - 6x)^5$.
(b). $6(10x^9 + 15x^2 - 6x)^5$.
(c). $6(x^{10} + 3x^5 - 6x)^5(10x^9 + 15x^4 - 6x)$.
(d). $10x^9 + 15x^4 - 6x^2$.

7. The critical numbers for the function $f(x) = x^3 - 9x$ are:
(a). $0, 3$, and $-3$.
(b). $\sqrt{3}, -\sqrt{3}$.
(c). 0.
(d). 3 and $-3$.

8. Which are the points of inflection in the following graph?




(a). Only $-1$ and 1.
(b). Only 0 and 1.
(c). Only 0.
(d). There are no inflection points in this graph.

9. Let $f$ have critical points at $-1, 0$, and 2. If $f''(x) = x^4 - 16x^2 + 9$, then $f$ has a relative maximum at:
(a). 0 and 2.
(b). $-1$ and 2.
(c). 0.
(d). None of the above answers.

10. As $x \to -\infty$, the function $f(x) = \frac{3x^2+15x-10}{4x^2-7}$ tends to
(a). 10/7.
(b). 3/4.
(c). 3.
(d). $-10$.

11. Let

$$y = f(x) = \frac{(x^2+x)^3}{x+2}.$$

Then $f'(x) =$
(a).

$$\frac{(x+2)[3(x^2+x)^2(2x+1)] - (x^2+x)^3}{(x+2)^2}.$$

(b).

$$3(x^2+x)^2(2x+1).$$

(c).

$$\frac{(x^2+x)^3 - (x+2)[3(x^2+x)^2(2x+1)]}{(x+2)^2}.$$

(d). None of the above answers.

Use the following information for the next two problems:
A supermarket expects to sell 10,000 boxes of a certain pet food in a year. The supermarket owner must pay \$3.00 for each box. There is a \$10.00 delivery charge for each delivery from the manufacturer, and there is a \$2.00 annual storage charge per box of pet food. Let $x$ be the number of boxes of pet food that the supermarket owner purchases from the manufacturer each time.

12. What are the annual storage costs for the dealer, in dollars?
(a). $3x$.
(b). $3x + 10$.
(c). $2x$.
(d). $x$.

13. What does it cost, in dollars, the supermarket owner each time she places an order for this pet food?
(a). $3x$.
(b). $3x + 10$.
(c). $5x$.
(d). $5x + 10$.

| 15 | 20 | 15 | 10 | 52 | 112 max. |
|---|---|---|---|---|---|
| A | B | C | D | m | Total |

NAME.....

Intuitive Calculus 11012 Examination 2 B
March 11, 2010 Richard M. Aron

**Directions: Please answer questions A, B, C, and D in the space provided.*Please write extremely neatly.* The rest of the Examination questions are to be answered on the "scan-tron" papers, *but you must show your work on this paper—even for the "scan-tron" part. Good luck!***

Product Rule: if $p(x) = f(x) \cdot g(x)$, then $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

Quotient Rule: if $q(x) = \frac{f(x)}{g(x)}$, then $q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$.

A. Let $f(x) = x^4 + 4x^3 + 4x^2$ on the interval $[-2, 1]$.

(a). Find all critical numbers of $f(x)$.

$y' = 4x^3 + 12x^2 + 8x$

$= 4x(x^2 + 3x + 2) = 4 \times (x+1)(x+2)$

0, -1, -2

(b). Classify the critical numbers found in part (a). (That is, explain whether a particular critical number corresponds to a relative maximum or a relative minimum of $f(x)$.)

$y'' = 12x^2 + 24x + 8$

$y''(0) = 8$, min

$y''(-1) = -4$, max

$y''(-2) = 8$, min

(c). Find the absolute extreme values of $f(x)$ on the closed interval $[-2, 1]$.

$y = x^4 + 4x^3 + 4x^2$

$y(-2) = 0$

$y(-1) = 1$

$y(0) = 0$

$y(1) = 9$

0 & 9

B. A car dealer can sell four cars per day at a price of \$24,000. She estimates that for each \$400 price reduction, she can sell one more car per day. What price should she charge to maximize her revenue R(x)? How many cars will she sell each day? [*Hint*: Let $x$ = the number of \$400 price reductions.]

Step 1. Let $p(x)$ be the price she charges for a car, after $x$ price reductions.

$p(x) = 24000 - 400x$

Step 2. Let $q(x)$ be the quantity of cars she sells in a day, after $x$ price reductions.

$q(x) = 4 + x$

Step 3. Calculate $R(x)$.

$R(x) = p(x)\,q(x) = (24000 - 400x)(4 + x)$

$= 96000 + 22400x - 400x^2$

Step 4. Maximize $R(x)$, explaining why your answer gives a maximum.

$R'(x) = 22400 - 800x, \quad R'' = -800 < 0$

$\therefore$ max

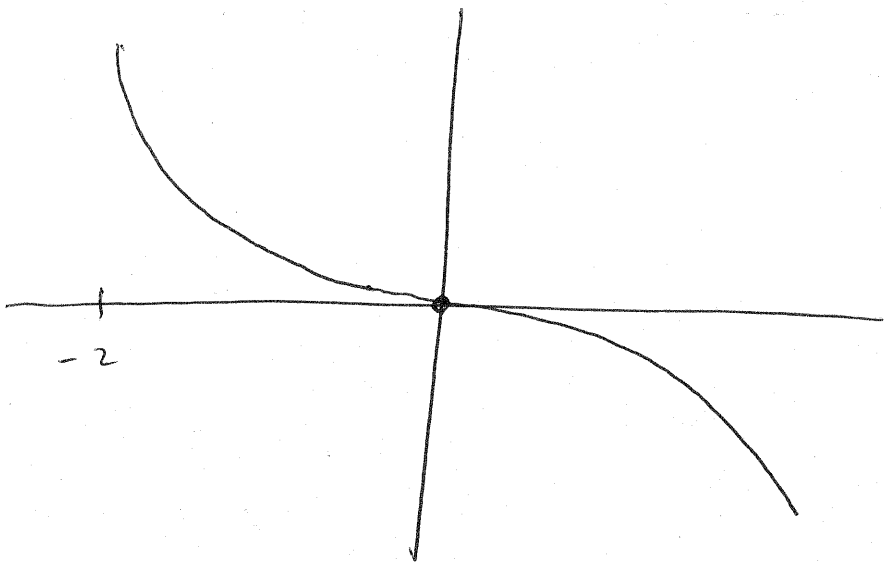
Step 5. What price should she charge to maximize her revenue? How many cars will she sell each day?

$x = \frac{22400}{800} = 28.$

price $p(28) = 412800$

$q(x) = q(28) = 32$

C. On the axes below, draw an *extremely clear, careful* sketch of the graph of a function $f$ which has the following properties: $f$ is continuous and differentiable, $f(0) = 0, f'(x) < 0$ for $x \in (0, 2)$, $f''(x) > 0$ on $(-2, 0)$ and $f$ is concave down on $(0, 2)$. Indicate any relative maxima or minima, and any inflection points.




D.(a). Evaluate. $f'$ and $f''$ for $f(x) = 4\sqrt{x^3} - 18\sqrt[3]{x^2}$. $= 4x^{3/2} - 18x^{2/3}$

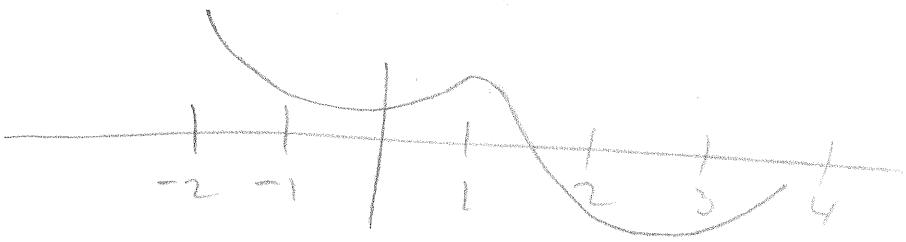
(a). $f'(x) =: 6x^{1/2} - 12x^{-1/3}$

(b). $f''(x) =: 3x^{-1/2} + 4x^{-4/3}$

**The rest of this Examination is to be done using the "scantron" sheet. Please write your work on this paper.**

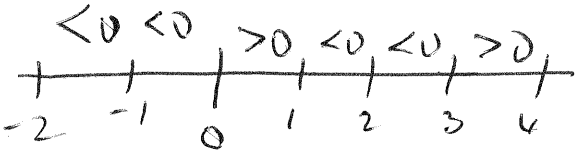
1. Let $f(x) = (3x-5)^8$. Then $f'(x) =$:
(a). $8(3x-5)^7$.
(b). $24(2x)^7$.
(c). $24(3x-5)^7$.
(d). None of the above answers.

2. Let $y = f(x)$ be a differentiable function on $[-2, 4]$, whose graph is given below.




Which of the following is the sign diagram for $f'$?

(a)




(b)

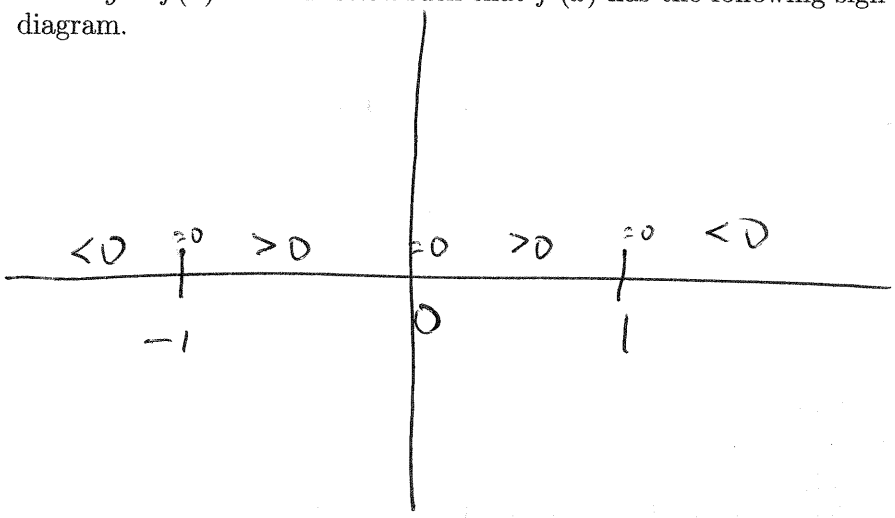
(c)

(d)

3. Let $y = f(x) = x^3 + 3x^2 - 9x$, where $x$ is in the closed interval $[-3, 4]$. Then the smallest and largest values that the function attains in this interval are:
(a). $-5$ and 76.
(b). 0 and $\infty$.
(c). $-5$ and 27.
(d). $-20$ and 32.

4. The point $x = 0$ is where the function $y = f(x) = x^4 - 8$ has:
(a). a relative minimum.
(b). a relative maximum.
(c). neither a relative maximum nor a relative minimum.
(d). Answer cannot be determined from the given information.

5. Let $y = f(x)$ be a function such that $f'(x)$ has the following sign diagram.

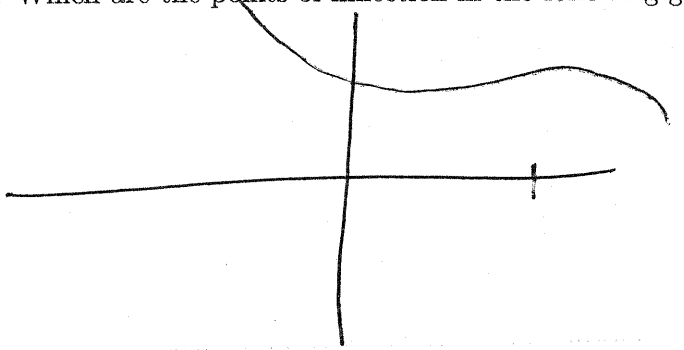



Which of the following is true? $f$ has:
(a). a relative maximum at $-1$.
(b). a relative maximum at $-1$ and a relative minimum at 0.
(c). relative maxima at $-1$ and at 1.
(d). a relative minimum at $-1$ and a relative maximum at 1.

6. The derivative of $y = f(x) = (x^{12} + 4x^4 - 3x^2 - 5)^4$ is:
(a). $12(x^{11} + 16x^3 - 9x)^3$.
(b). $4(x^{12} + 4x^4 - 3x^2 - 5)^3(12x^{11} + 16x^3 - 6x)$.
(c). $4(x^{12} + 4x^4 - 3x^2 - 5)^3$.
(d). $12x^{11} + 16x^3 - 6x$.

7. The critical numbers for the function $f(x) = x^3 - 25x$ are:
(a). $5\sqrt{3}$ and $-5\sqrt{3}$.
(b). $0, 5$, and $-5$.
(c). 5 and $-5$.
(d). 0.

8. Which are the points of inflection in the following graph?



(a). Only $-1$ and 1.
(b). Only 0 and 1.
(c). Only 1.
(d). There are no inflection points in this graph.

9. Let $f$ have critical points at $-1, 0$, and 1. If $f''(x) = x^3 - 8x^2 + 2$, then $f$ has a relative maximum at:
(a). 0 and 1.
(b). $-1$ and 1.
(c). 0.
(d). None of the above answers.

$f''(0) = 2 > 0$

$f''(1) = -5 < 0$ max

$f''(-1) = -7 < 0$ max

10. As $x \to -\infty$, the function $f(x) = \frac{4x^3+5x-1}{2x^2+3}$ tends to
(a). 2.
(b). $-1/3$.
(c). $-1$.
(d). 4.

11. Let

$$y = f(x) = \frac{(x^2-x)^3}{x+1}.$$

Then $f'(x) =$

(a).

$$\frac{(x^2-x)^3 - 3(x+1)(x^2-x)^2(2x-1)}{(x+1)^2}.$$

(b).

$$3(x^2-x)^2(2x-1).$$

(c).

$$\frac{3(x+1)(x^2-x)^2(2x-1) - (x^2-x)^3}{(x+1)^2}.$$

(d). None of the above answers.

Use the following information for the next two problems:
A supermarket expects to sell 16,000 boxes of a certain pet food in a year. The supermarket owner must pay \$2.00 for each box. There is a \$20.00 delivery charge for each delivery from the manufacturer, and there is a \$3.00 annual storage charge per box of pet food. Let $x$ be the number of boxes of pet food that the supermarket owner purchases from the manufacturer each time.

12. What are the annual storage costs for the dealer, in dollars?
(a). $3x/2$.
(b). $3x + 20$.
(c). $2x$.
(d). $3x$.

13. What does it cost, in dollars, the supermarket owner each time he places an order for this pet food?
(a). $3x$.
(b). $3x + 20$.
(c). $2x$.
(d). $2x + 20$.