① $\int_0^t s\,dW(s) = tW(t) - \int_0^t W(s)\,ds$ ①

Indeed

$$I_n = \sum_{k=0}^{n-1} \frac{tk}{n}\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]$$

$$\overset{\text{by hint}}{=} \underbrace{\sum_{k=0}^{n-1}\left[\frac{t(k+1)}{n} W\left(\frac{t(k+1)}{n}\right) - \frac{tk}{n} W\left(\frac{tk}{n}\right)\right]}_{\text{this is a telescopic sum}} - \sum_{k=0}^{n-1} W\left(\frac{t(k+1)}{n}\right)\underbrace{\left[\frac{t(k+1)}{n} - \frac{tk}{n}\right]}_{= \frac{t}{n}}$$

$$= \left[t\,W(t) - 0\,W(0)\right] - \underbrace{\sum_{k=0}^{n-1} W\left(\frac{t(k+1)}{n}\right)\frac{t}{n}}_{\text{Riemann sum}}$$

$$= tW(t) - \int_0^t W(s)\,ds$$

② $\int_0^t W^2(s)\,dW(s) = \frac{1}{3}W^3(s) - \int_0^t W(s)\,ds$

$$I_n = \sum_{k=0}^{n-1} W\left(\frac{tk}{n}\right)^2\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]$$

$$\overset{\text{by hint}}{=} \frac{1}{3}\sum_{k=0}^{n-1}\left[W\left(\frac{t(k+1)}{n}\right)^3 - W\left(\frac{tk}{n}\right)^3\right]$$

$$- \sum_{k=0}^{n-1} W\left(\frac{tk}{n}\right)\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^2$$

$$- \frac{1}{3}\sum_{k=0}^{n-1}\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^3$$

Now $\frac{1}{3}\sum_{k=0}^{n-1}\left(W^3\left(\frac{t(k+1)}{n}\right) - W^3\left(\frac{tk}{n}\right)\right)$

is a telescopic sum, hence equal the last term minus

first i.e. $W^3(t) - W^3(0)$

$$\sum_{k=0}^{n-1} W\left(\frac{tk}{n}\right)\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^2$$

$$= \sum_{k=0}^{n-1} W\left(\frac{tk}{n}\right)\left\{\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^2 - \left(\frac{t(k+1)}{n} - \frac{tk}{n}\right)\right\}$$

$$+ \underbrace{\sum_{i=0}^{n-1} W\left(\frac{tk}{n}\right)\left[\frac{t(k+1)}{n} - \frac{tk}{n}\right]}$$

this is a Riemann sum that converges to $\int_0^t W(s)ds$

We compute now the limit in L^2 of

$$\sum W\left(\frac{tk}{n}\right)\left\{\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^2 - \left(\underbrace{\frac{t(k+1)}{n} - \frac{tk}{n}}_{=\frac{t}{n}}\right)\right\}$$

Take now the limit in L^2 i.e. square everything and then take expectation. Since the 2 terms in the product are independent ($W\left(\frac{tk}{n}\right) \in \mathcal{F}_{\frac{tk}{n}}$ and $W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)$ is independent on $\mathcal{F}_{\frac{tk}{n}}$) we get

$$\sum_{k=0}^{n-1} \underbrace{E\left(W^2\left(\frac{tk}{n}\right)\right)}_{=\frac{tk}{n}} E\left\{\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^2 - \frac{t}{n}\right\}$$

$$= \sum_{k=0}^{n-1} \frac{tk}{n}\left\{\left[E\left(W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right)^2\right] - \frac{t}{n}\right\}$$

$$= \sum_{k=0}^{n-1} \frac{tk}{n}\left(\frac{t}{n} - \frac{t}{n}\right) = 0$$

For the last term $\displaystyle\sum_{k=0}^{n-1}\left[W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right]^3$

When we square and we take expectations we get $\displaystyle\sum_{k=0}^{n-1} E\left(\left(W\left(\frac{t(k+1)}{n}\right) - W\left(\frac{tk}{n}\right)\right)^6\right)$

$$= 6\sum_{k=0}^{n-1}\left(\frac{t}{n}\right)^3 = \frac{6t^3}{n^2} \to 0$$

Putting everything together we get the desired result.