M 4-5 1:45 : 2:15.
W 4-5

??

$\mathcal{F} = \{ A \mid A \subset \Omega \}$

1
2
3

$\mathcal{P}(\Omega)$ = power set.

Is $P(\Omega)$ a field?

1. $\Omega \in P(\Omega)$
2. If $A \in P(\Omega) \Rightarrow A^c \in P(\Omega)$?
3. $A_1, A_2, \ldots \in \Omega \Rightarrow \cup A_i \in P(\Omega)$

Ex: $\mathcal{F} = \{ \Omega , \phi \}$

$\Omega = \{a, b\}$

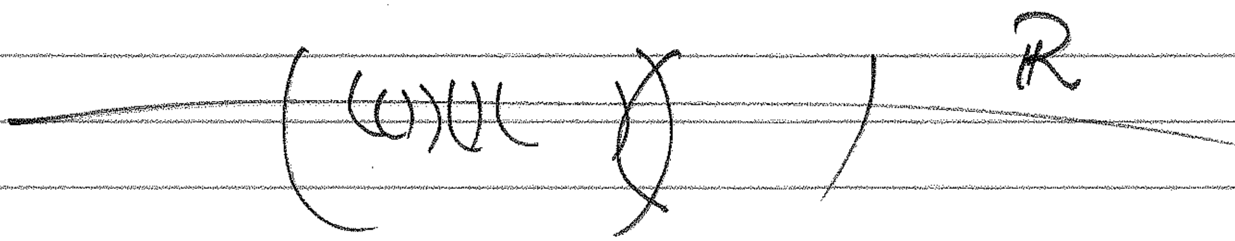
$\mathcal{F} = \{\{a,b\}, \phi\}$

$\mathcal{F} = \{\{a,b\}, \phi, \{a\}, \{b\}\} = \mathcal{P}(\Omega)$

A, B

$A \cup B$

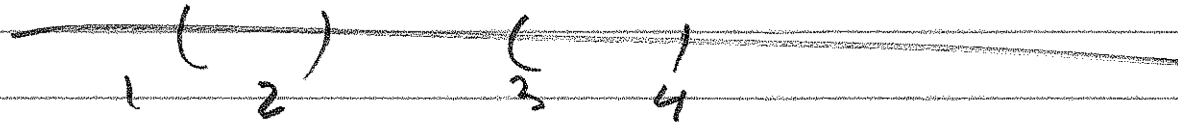
$(A^c \cup B^c)^c = A \cap B$



$\Omega = \mathbb{R}$

intervals $(a,b) \subset \Omega$

$\mathcal{B}(\Omega) = \{ (a,b), (a,b) \subset \mathbb{R}$



$(1,2) \cup (3,4)$

$\{0\}$ $(1,2)^c = (-\infty, 1] \cup [2, \infty)$

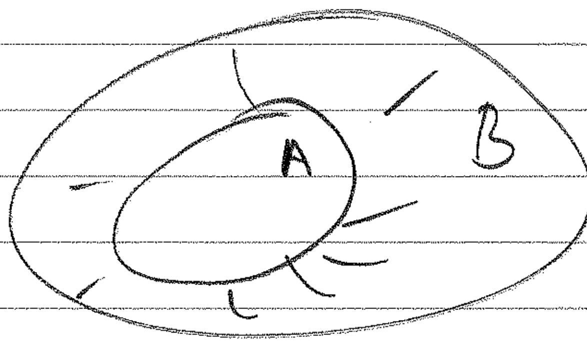
$\{0\} \subset \mathcal{B}(\mathbb{R})$?

$\{0\} = [0,1) \cap (-\infty, 0]$

$\{0,1\} \subset \mathcal{B}(\mathbb{R})$

$= \{[0,1) \cap (-\infty, 0]\} \cup \{[1,2) \cap (0,1]\}$

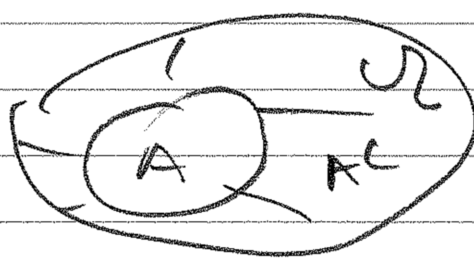
(1) $A \subset B \quad P(A) \leq P(B)$



$B = A \cup (B \setminus A)$

$P(B) = P(A \cup (B \setminus A)) = P(A) + \underbrace{P(B \setminus A)}_{\geq 0}$

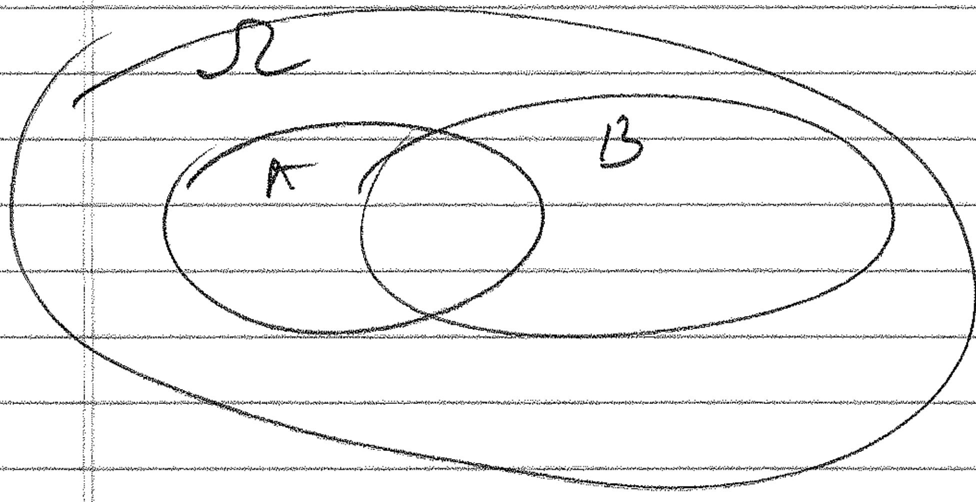
(2) $P(A^c) = 1 - P(A)$



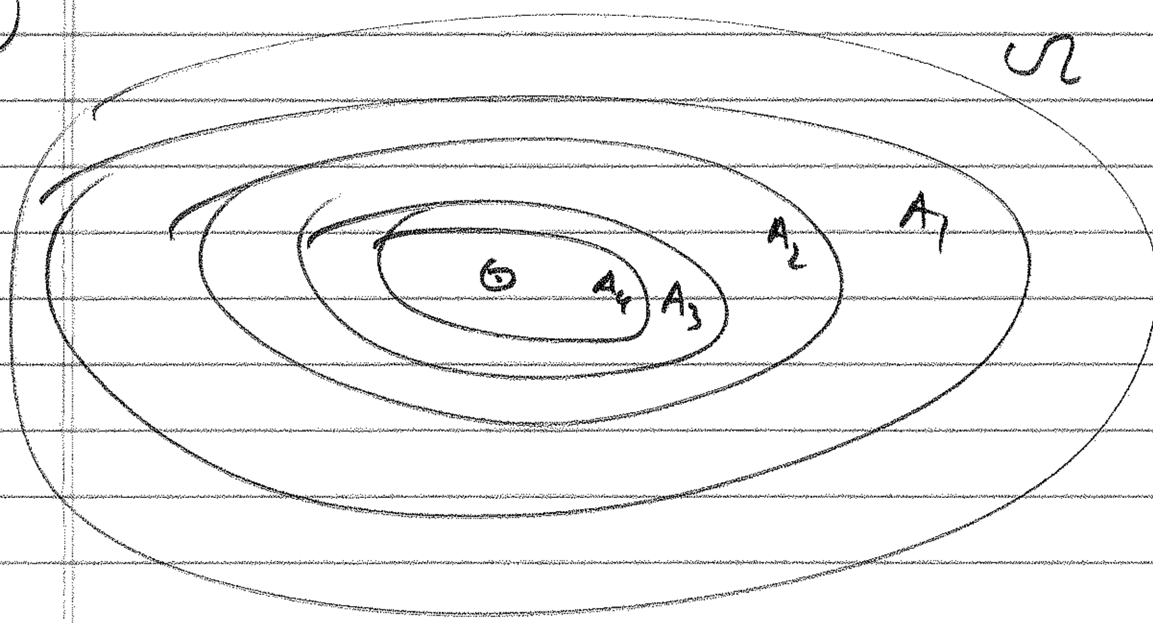
$\Omega = A \cup A^c$

$1 = P(A) + P(A^c)$

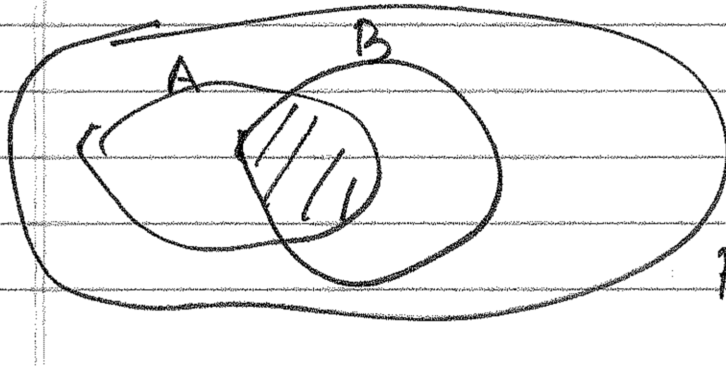
(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



(4)



$P(A) = P(A|B)P(B) + P(A|B^c)P(B^c)$



$A = (A \cap B) \cup (A \cap B^c)$

$P(A) = P(A \cap B) + P(A \cap B^c)$

$$P(A|B)\,P(B) + P(A|B^c)\,P(B^c)$$

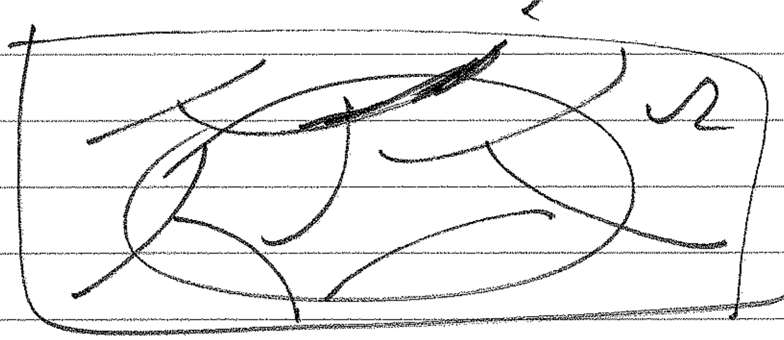
$$= \frac{P(A \cap B)}{\cancel{P(B)}} \cdot \cancel{P(B)} + \frac{P(A \cap B^c)}{\cancel{P(B^c)}} \cdot \cancel{P(B^c)}$$

$$= P(A \cap B) + P(A \cap B^c) = P(A)$$

$$P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}$$

$$=$$

$$\frac{P(A \cap B)}{\cancel{P(B)}} \cdot \frac{\cancel{P(B)}}{P(A)} = \frac{P(A \cap B)}{P(A)}$$



$A_1 \quad A_2 \quad A_3 \quad A_4 \quad A_5 \quad A_6$

$$P(A_1 \cap A_2 \cap \cdots \cap A_6) = P(A_1)P(A_2) \cdots P(A_6)$$

$$P(A_1 \cap A_2 \cdots \cap A_5) = P(A_1)P(A_2) \cdots P(A_5)$$

$$P(A_2 \cap A_3 \cap \cdots A_6) = P(A_2)P(A_3) \cdots P(A_6)$$

$$\vdots$$