SAMPLE PROBLEMS FOR THE QUIZ

1. There are three boxes, A,B, and C, such that box A has 10 red and 16 green balls, box B has 11 red and 15 green balls, and box C has 12 red and 14 green balls. A box is selected at random, and from the selected box, a ball is drawn at random.

   Answer the following:

   (a) State a form of the Theorem of Total Probability.

   (b) What is the probability that the drawn ball is red?

   (c) If the selected ball is red, what is the probability that it came from box A?

2. A point is selected at random from the rectangle $[0, 2] \times [0, 1]$. What is the probability that the point chosen falls below the curve $f(t) = 2t - t^2$ (Hint: draw the regions).

3. Let X and Y be two independent random variables. Find the mean and variance of the random variable $Z = 3X - Y$ if:

   (a) X has an exponential distribution with mean 2 and variance 4, while Y has a Poisson distribution with $E(Y) = \text{Var}(Y) = 3$.

   (b) They are both normally distributed, with means $\mu_X = 1$, $\mu_Y = 5$, and standard deviations $\sigma_X = 3$ and $\sigma_Y = 3$. What is the name of the distribution of Z in this case?

4. The joint density of two random variables X, Y is given by

$$f(x, y) = \begin{cases} e^{-x}, & 0 \leq y \leq x < \infty \\ 0, & \text{otherwise.} \end{cases}$$

(a) Compute the marginal density $f_X(x)$ and identify the corresponding distribution.

(b) Give the conditional density $f_{Y|X=x}(y)$. Be very careful about saying where your density is zero.

(c) Compute the conditional expectation $E(Y|X = x)$.

5. Random variables X and Y have the following joint probability mass function:

$$p_{XY} = \begin{cases} c\dfrac{e^{-5}5^x}{x!}(1+y), & \text{for } x = 0, 1, \ldots, \ y = 0, 1, 2. \\ 0, & \text{otherwise} \end{cases}$$

for some appropriate c.

(a) What is the appropriate value for c?

(b) What is the probability that $X \geq 1$ or $Y = 1$?

(c) What is $P_{X|Y}(x|2)$?

(d) What is $E(X)$?

(e) What is $\text{Var}(Y)$?

(f) Are X and Y independent?

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6. For each of the following, answer **TRUE** or **FALSE**. Give a brief justification for your answer.

(a) ☐ TRUE ☐ FALSE Given events $A \subset B$, we always have $P(A|B) \geq P(A)$.

(b) ☐ TRUE ☐ FALSE $E(1/Y) = 1/E(Y)$, for any random variable Y.

(c) ☐ TRUE ☐ FALSE $\text{Var}(X) \geq 0$ for any random variable X.