Calculus I - Test 1 Review - Fall 2017 - Dr. Smithies

Test One is **Friday September 22nd**. Unless you have a documented emergency on that date, you may incur a late penalty on a make up test. So please make every effort to be in the class on time and ready to work on this day.

If you have a documented reason why you would like to arrange your test with Student Access Service, you need to contact them. Their website is http://www.kent.edu/sas. Please let me know if you are testing through SAS since I need to email them the test.

Test One will cover sections 1.1 - 1.6 and 2.1-2.5 of our book. Sections 1.1, 1.2, and much of section 1.6 are prerequisites for this class. You should review these sections on your own. The assigned homework for test one material was: the problems with solutions in the back of your book from the problem sets:

Section	**Topic**	**Problems**	**Section**	**Topic**	**Problems**
1.3	Limits	1-18	2.1	Derivatives	1-36
1.4	Calculating Limits	1-44	2.2	Derivatives	1-27
1.5	Continuity	1-42	2.3	Calculating Derivatives	1-52
1.6	Infinity and Limits	1-33	2.4	Product/Quotient Rule	1-45
			2.5	Chain Rule	1-48

These homework problems, with the worked examples from class and our book are the material to review for the test. **Definitions** are important potential test questions.

Prerequisites

Section(1.1) A function is a rule or correspondence which assigns to each element x of its domain exactly one element $f(x)$ in its range. The graph of a function $f(x)$ is the set of all ordered pairs $(x, f(x))$ such that x lies in the domain of $f(x)$. No vertical line crosses the graph of a function more than once.

Section(1.1) In general, the domain of $f(x)$ is all real numbers where the function is defined. You calculate the domain by excluding any real numbers which create (a) division by zero; or (b) an even root of a negative number. The range of $f(x)$ is the set of all values the function takes on.

Section(1.1) Given a function in any form - table, graph or formula, be able to evaluate the function at any given number, variable or expression.

Example: Let $f(t) = (t-4)^2 + \sqrt{t+2}$. Find $f(3)$ and $f(3x+2)$. Here $f(3) = (3-4)^2 + \sqrt{3+2} = 1 + \sqrt{5}$, and $f(3x+2) = (3x+2-4)^2 + \sqrt{3x+2+2} = (3x-2)^2 + \sqrt{3x+4}$.

Section(1.2) Be able to sketch the graph of $y = x^n$ for $n = \cdots, -2, -1, 0, 1, 2, \cdots$, $y = \sqrt{x}$, $y = |x|$. Also know the graphs of the six trigonometry functions $\sin(x)$, $\cos(x)$, $\tan(x)$, $\csc(x)$, $\sec(x)$, $\cot(x)$.

Section(1.2) The graphing basics that you should know include the following:

The x *intercept(s)* are the points $(x_1, 0), \cdots, (x_n, 0)$ where $x_1, \cdots, x_n$ are found by solving $f(x) = 0$ for x. The y *intercept* is the points $(0, f(0))$ where you get the y coordinate by plugging 0 into your function. Know the various ways a graph will be shifted or scaled. Namely, the graph of $cf(x+a)+b$ is a scale by c of the graph of $f(x)$ shifted left a units and up b units.

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Chapter 1

Recall the following definitions of one-sided and two-sided limits:

$\lim_{x\to a^-} f(x) = c$ means that as x get arbitrarily close to a but remains less than a, the value of the function $f(x)$ get arbitrarily close to c. Similarly, $\lim_{x\to a^+} f(x) = c$ means that as x get arbitrarily close to a but remains larger than a, the value of the function $f(x)$ get arbitrarily close to c. Finally, $\lim_{x\to a} f(x) = c$ means that both one-sided limits exist and they are equal, i.e., $\lim_{x\to a^-} f(x) = c$ and $\lim_{x\to a^+} f(x) = c$.

When computing $\lim_{x\to a} f(x)$ start by plugging a into $f(x)$. If you get a finite number, this will generally be the limit. Otherwise,
If $f(a) = \frac{\text{positive number}}{0}$ then $\lim_{x\to a} f(x) = \infty$.
If $f(a) = \frac{\text{negative number}}{0}$ then $\lim_{x\to a} f(x) = -\infty$.
If $f(a) = \frac{0}{0}$, $\frac{\infty}{\infty}$ or $\infty - \infty$ then $\lim_{x\to a} f(x)$ is UNDETERMINED. That is, you have to do some algebra or computation to rewrite $f(x)$ in order to remove the division by zero.

A continuous function looks like a function which can be drawn without lifting the pencil. By definition, $f(x)$ is continuous at $x = a$ means $\lim_{x\to a} f(x) = f(a)$. More precisely, it means that (i) the function $f(x)$ is defined at a; (ii) the limit of $f(x)$ as x approaches a exists; and (iii) this limit equals the function value of $f(x)$ at a.

All polynomials and the sine and cosine functions are continuous at each a in the real numbers. Rational functions (ratio of polynomials) and the other four trigonometry functions are continuous everywhere they are defined. That is, at every real number except those that create a division by zero.

Any product, sum, difference and quotient (which doesn't create division by zero) of continuous functions is a continuous function. Powers and roots of continuous functions are continuous functions, as long as they do not create the even root of a negative number.

A function f has the vertical line $x = c$ as an asymptote exactly when $f(c)$ has denominator zero and numerator which is not zero.
If $\lim_{x\to c^-} f(x) = -\infty$ the function drops down the line $x = c$ on the left of this line.
If $\lim_{x\to c^-} f(x) = \infty$ the function climbs up the line $x = c$ on the left of this line.
If $\lim_{x\to c^+} f(x) = -\infty$ the function drops down the line $x = c$ on the right of this line.
If $\lim_{x\to c^+} f(x) = \infty$ the function climbs up the line $x = c$ on the right of this line.

If $f(x) = \frac{a_n x^n + \cdots + a_1 x + a_0}{b_k x^k + \cdots + b_1 x b_0}$ has the form of one polynomial divided by another there are 3 possibilities. If $n > k$ there may be a slant asymptote but there is no horizontal asymptote; if $n = k$ the line $y = \frac{a_n}{b_k}$ is a horizontal asymptote; if $n < k$ the line $y = 0$ (i.e., the x-axis) is a horizontal asymptote. A function can cross a horizontal asymptote. A horizontal asymptote describes the behavior of the function at the left edge ($\lim_{x\to -\infty} f(x)$) and the right edge ($\lim_{x\to\infty} f(x)$) of the graph.

Generally speaking, the graph of a function $f(x)$ has a vertical asymptote $x = a$ when plugging a into $f(x)$ creates a zero in the denominator but not in the numerator; it has a hole at (a, b) when plugging a into $f(x)$ creates a zero in the denominator and in the numerator, here the y-coordinates of the hole is $b = \lim_{x\to a} f(x)$; it has a jump at $x = a$ when $f(x)$ is defined piece-wise, with a formula change at $x = a$.

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Chapter 2

• The *difference quotient* of $f(x)$ at $x = a$ is

$$\frac{f(a+h) - f(a)}{h}.$$

It is the slope of the secant line between the points $(a, f(a))$ and $(a + h, f(a + h))$ on the graph of $f(x)$.

• The limit of the difference quotient as h goes to 0 is the *derivative* of $f(x)$ at a, $f'(a)$. That is,

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

The slope of the tangent line to the graph of $f(x)$ at $(a, f(a))$ is the derivative at a, $f'(a)$. The equation for this tangent line to the graph of $f(x)$ at $(a, f(a))$ is

$$y - f(a) = f'(a)(x - a).$$

Remark: The physical interpretations of difference quotients and derivatives are as follows: The difference quotient of $f(x)$ at a is the *average rate of change* of $f(x)$ over the interval $[a, a + h]$; The derivative to $f(x)$ at a is the *instantaneous rate of change* of $f(x)$ at a. But Test One will not include these interpretations.

• Using the definition of the derivative, we established *the power rule*

$$\frac{d}{dx}[x^n] = nx^{n-1}.$$

This rule holds for any real number n. It is useful to recall that $x = x^1$, $1 = x^0$, $\sqrt{x} = x^{\frac{1}{2}}$, $\frac{1}{x} = x^{-1}$.

• We also showed that the derivative is *linear*. This means that the derivative distributes across sums and differences and that constant factors are carried along. More precisely,

$$[cf(x)]' = cf'(x) \text{ for any constant } c$$

and

$$[f(x) + g(x)]' = f'(x) + g'(x).$$

The linear property of the derivative which is described above, is used to compute derivatives. For example, if $f(x) = 7x^2 - \frac{3}{x}$ then

$$f'(x) = 7(x^2)' - 3(\frac{1}{x})' = 7(2x) - 3(\frac{-1}{x^2}) = 14x + \frac{3}{x^2}$$

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Chapter 2

• **Product, Quotient and Chain Rules** When a function is formed by multiplying, dividing or composing more basic functions, we can calculate its derivative in terms of these basic functions. Specifically,

Function	Derivative
$y = u(x)v(x)$	$y' = u'(x)v(x) + v'(x)u(x)$
$y = \frac{N(x)}{D(x)}$	$y' = \frac{N'(x)D(x) - D'(x)N(x)}{[D(x)]^2}$
$y = f \circ g(x) = f(g(x))$	$y' = f'(g(x))g'(x)$

You should know these commonly used derivatives
$(\sin(x))' = \cos(x)$
$(\cos(x))' = -\sin(x)$
$(\sec(x))' = \tan(x)\sec(x)$
$(x^n)' = nx^{n-1}$
$(c)' = 0$ (c constant.)
$(\csc(x))' = -\cot(x)\csc(x)$
$(\cot(x))' = -\csc^2(x)$
$(\tan(x))' = \sec^2(x)$