

Counting number of ways to do a task.

1) Break the problem up into **non-overlapping** subtasks

Example: Count ways to make a license plate of the form ABC123 (three letters followed by three numbers)

Task 1: Choose the first letter

Task 2: Choose the 2nd letter

Task 3: Choose the 3rd letter

Task 4: choose 1 st number

Task 5: choose 2 nd number

Task 6: choose 3 rd number

2) Count the number of ways to do each subtask.

In our example, each of the first 3 subtasks have 26 ways. The last three subtasks have 10 ways each (digits 0 through 9).

3) If the original task is completed by doing each subtask, (must do ALL subtasks to complete), then the total ways to complete the original task is the product of the subtasks. (multiply the subtasks)

The answer to our example is $26 \times 26 \times 26 \times 10 \times 10 \times 10$
i.e. $26^3 \times 10^3 = 17,576,000$

How many ways to make the license plate of the form ABC123 if each character must be distinct (no repeated letters or numbers)?

1st task: 26 ways (pick any letter)

2nd task: 25 ways (pick any letter not the same as the 1st)

3rd task: 24 ways

4th task: 10 (pick a number 0-9)

5th task: 9 (pick a different number)

6th task: 8

Total:

$$26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11,232,000$$

4) If the task is completed by doing exactly one of the subtasks, then the number of ways to do the original task is the sum of the subtasks (add the subtasks together)

Example: A password character must be a digit, an upper or lower case letter, or one of the special characters +, *, !, or ?.

- a) How many password characters are possible?
- b) How many 4 character passwords are possible?
- c) How many password characters are possible?

Task= choose a character

Subtask 1: choose a digit

OR

Subtask 2: choose a lower case letter

OR

Subtask 3: choose a capital letter

OR

Subtask 4: choose a special character

-Only need to complete ONE subtask to complete original task,
so we need to add the subtasks.

Total ways to pick a character:

$$10+26+26+4 = 66$$

How many 4 character passwords are possible?

Recall: 66 possible characters

Subtask 1: pick 1st character

AND Subtask 2: pick 2nd character

AND Subtask 3: pick 3rd character

AND Subtask 4: pick 4th character

Since we need to complete ALL subtasks to complete the original task, we need to multiply the subtasks.

Total ways to make a password:

$$66^4 = 18,974,736$$

Comments:

OR = add, AND = multiply

(do task 1 OR task 2)

(do task 1 AND task 2)

How many legs does a horse have?

Answer: 8

Proof: two on the right, two on the left, two in the front, two in the back, so $2+2+2+2=8$

(example of counting mistake)

These subtasks overlapped. Make sure your subtasks do not overlap! (do not count the same thing more than once)

Notation:

$$N! = 1 \cdot 2 \cdot 3 \cdot 4 \cdots (N-1) \cdot N$$

$$0! = 1, \text{ and } 1! = 1, 3! = 1 \times 2 \times 3$$

How many ways to arrange 5 books on a shelf:

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$