

Review For Test Two

Our second test is Wednesday October 30th. It is closed book and note. You will need your own calculator (not a cell phone or computer); You cannot share calculators. The test will cover voting methods and theory. Specifically, you must know how to use each of our 4 voting methods to find a winner or rank the candidates. You should be able to use counting methods pertaining to voting and be able to describe how each of our voting methods satisfies or violates a given fairness criterion. The main ideas are summarized below.

Formula: Sum whole numbers from 1 to N .

The total when you add the whole numbers from 1 to N is

$$1 + 2 + 3 + \cdots + N = \frac{N(N + 1)}{2}.$$

For example, $1 + 2 + 3 + 4 + 5 = 15$, but instead of adding you could get that from using $N = 5$ in this formula. This gives you the total $\frac{5(6)}{2} = \frac{30}{2} = 15$.

Formula: The number of pairwise comparisons in an election with N candidates is $\frac{N(N-1)}{2}$. This is the sum $1 + 2 + 3 + \cdots + (N - 1)$.

Formula: The number of different ballots which can be constructed in an election with N candidates is $N! = N(N - 1)(N - 2) \cdots (2)(1)$. This is the multiplication rule with Subtask 1 = Choose candidate to rank 1st, through Subtask N = Choose candidate to rank last.

Formula: The number of different ballots which can be cast in an election with N candidates and V voters is the smaller of the two numbers $N!$ and V . You can construct $N!$ ballots but you cannot have more ballots cast than there are voters.

Voting Methods

A voting method is a way to choose a winner when *preference ballots* are cast in an election with 3 or more candidates. We also use voting methods to give an *extended ranking* or a *recursive ranking* of the candidates. The methods which will be on our test are:

- (1) Plurality: Count the number of first place votes each candidate receives. The winner(s) is the candidate with most first place votes.
- (2) Pairwise Comparison: Compare every two candidates head-to-head and assign points as follows: Give 1 point for each win, $\frac{1}{2}$ point each for each tie and 0 points for each loss. The winner(s) is the candidate with the most points.
- (3) Borda: If the election has N candidates give each candidate N points for each first place vote, $N - 1$ points for each second place vote, \cdots , and 1 point for each last place vote. Now total the points for each candidate. The winner(s) is the candidate with the most points.
- (4) Plurality-with-elimination: Count the number of first place votes each candidate has. Eliminate the candidate(s) with the fewest first place votes and transfer the eliminated candidate's first place votes to the next most preferred candidate. Repeat until some candidate has a majority (more than half) of the first place votes. This candidate wins.

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Voting Theory

A *fairness criterion* is a principle which describes an election circumstance and identifies the candidate who should win in this circumstance. We studied four fairness criteria; they are described below. A given voting method *satisfies* a given fairness criterion if it always selects the winner that the criterion mandates. If a given voting method does not satisfy a given fairness criterion, then we say the method *violates* the criterion. *Arrow's Impossibility Theorem* says that no voting method can satisfy all four of the fairness criteria. See also the Voting Fairness handout which lists which fairness criteria each of our four voting methods satisfy.

- (1) Majority Criterion - If Candidate X has more than half of the available first place votes, then Candidate X should win.
- (2) Condorcet Criterion - If Candidate X wins in head-to-head comparison against every opponent, then Candidate X should win.
- (3) Monotonicity Criterion - If Candidate X wins an election and in a re-vote only Candidate X has improved support, then Candidate X should win the re-vote.
- (4) Independence of Irrelevant Alternatives - If Candidate X wins an election and one or more losing candidates drops out of the election and is removed from the preference schedule then Candidate X should still win when the votes are re-tabulated.

Ranking Methods

There are two common ways to rank the candidates. In the ranking method we studied, you interpret the voting method calculation to rank the candidates. In Plurality, Borda and Pairwise Comparison the candidates are ranked most points to fewest points. In Plurality-with-elimination the first candidate eliminated is ranked lowest and so on until the winner is ranked 1st.

Ties

Any voting method could potentially yield a tie. People often agree in advance to resolve ties by head-to-head comparison, most 1st place votes, or fewest last place votes. Some ties are unresolvable by any method other than chance (e.g., flip of a coin, card from a deck, etc.) An unresolvable tie happens when all N candidates have exactly V 1st, 2nd, 3rd, \dots , N th place votes. For example, every voting method will end in a 3-way tie for this preference schedule, even though the number of voters (21) is odd.

Number of Votes	7	7	7
1st Choice	A	B	C
2nd Choice	B	C	A
3rd Choice	C	A	B