Excursions - Math 11008-002 - Fall 2019 - Dr. Smithies
Voting Theory - Fairness of Voting Methods.

A *voting method* is a way of selecting a winner in an election. A *fairness criterion* is a principle which describes an election circumstance and identifies the candidate who should win in this circumstance. A given voting method *satisfies* a given fairness criterion if it always selects the winner that the criterion mandates. If a given voting method does not satisfy a given fairness criterion, then we say the method *violates* the criterion.

You *prove* that a given voting method satisfies a given fairness criterion by explaining how whenever the criterion mandates that a certain candidate should win, this voting method will always choose this candidate as the winner. You *prove* that a given voting method violates a given fairness criterion by giving an example of an election in which the fairness criterion mandates that a certain candidate win but the voting method fails to select this candidate.

In Chapter 1, we study 4 voting methods (Plurality, Pairwise Comparison, Borda, Plurality-with-elimination) and 4 fairness criteria (Majority, Condorcet, Monotonicity and IIA, which is Independence of Irrelevant Alternatives). There are other voting methods and other fairness criterion which can be studied but we will focus on only these four. The relationships between these methods and criteria are summarized in the following table.

Satisfies/Violates	Majority	Condorcet	Monotonicity	Independence
Plurality	S	V	S	V
Pair Compare	S	S	S	V
Borda	V	V	S	V
Plur-w-Elim	S	V	V	V

Example 1 Prove that the Plurality method satisfies the Majority criterion.

The Majority Criterion states that if a candidate has more than half of the available first place votes, then this candidate should win the election. Plurality chooses the candidate with the most first place votes as the winner. If a candidate has more than half of the available first place votes, then this candidate must have more first place votes than any other candidate. Consequently, whenever an election has a Majority candidate, the Plurality method will always select this candidate as the winner.

Example 2 Prove that the Borda method violates the Majority criterion.

Consider the following Preference Schedule and Borda point totals.

Number of Votes	6	3	2
1st Choice	A	B	B
2nd Choice	B	C	D
3rd Choice	C	D	C
4th Choice	D	A	A

$$\begin{aligned} \text{A} &= 4(6) + 3(0) + 2(0) + 1(5) = 29 \\ \text{B} &= 4(5) + 3(6) + 2(0) + 1(0) = 38 \\ \text{C} &= 4(0) + 3(3) + 2(8) + 1(0) = 25 \\ \text{D} &= 4(0) + 3(2) + 2(3) + 1(6) = 18. \end{aligned}$$

Thus, candidate B wins this election by the Borda method. However, candidate A has 6 of the 11 first place votes. So, the Majority criterion says that A should win. This proves that the Borda method violates the Majority criterion.