

Explorations in Modern Math - Fall 2019 - Dr. Smithies
Weighted Voting Practice Test

Practice Problems for Weighted Voting.

1. In the weighted voting system $[30 : 12, 7, 4, 4, 3, 3, 1, 1]$ how many players are there? what is the quota? what is the total number of votes? what percent approval does the quota represent?
2. In the weighted voting system $[9 : 11, 4, 2]$ which players are dictators, have veto power or are dummies? Same question for $[14 : 7, 7, 4]$ and $[10 : 5, 4, 2]$
3. In the weighted voting system $[q : 12, 7, 5, 1, 1]$ what is the smallest possible value of the quota q ? what is the smallest quota which makes players P_4 and P_5 dummies? What is the smallest quota which gives P_1 veto power?
4. In the weighted voting system $[15 : 12, 8, 4, 2]$ what is the minimum percentage of votes needed to pass a motion? what should the quota be to require 75 percent approval for a motion to pass?
5. In $[35 : 20, 15, 10, 5]$ what is the weight of the coalition $\{P_2, P_3, P_4\}$? which players are critical in the coalition $\{P_1, P_2, P_3, P_4\}$? what are the winning coalitions and what are their weights? How many times is each player critical? what is the Banzhaf power distribution of the system?
6. In the generic weighted voting system $[q : w_1, w_2, \dots, w_N]$ what is mathematically equivalent to the statement that P_1 is a dictator? what condition is mathematically equivalent to the statement that P_1 has veto power?
7. In a certain weighted voting system there are 5 players and the winning coalitions are all coalitions with 3 or more players one of which is P_1 what are the winning coalitions? How many times is each player critical? what is the Banzhaf power distribution of the system?

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Answers to Practice Problems for Weighted Voting.

1. In the weighted voting system $[30 : 12, 7, 4, 4, 3, 3, 1, 1]$ how many players are there? (8) what is the quota? (30) what is the total number of votes? (35) what percent approval does the quota represent? ($\frac{30}{35} = 85.7\%$ or just 85%.)
2. In the weighted voting system $[9 : 11, 4, 2]$ which players are dictators, have veto power or are dummies? Same question for $[14 : 7, 7, 4]$ and $[10 : 5, 4, 2]$ In $[9 : 11, 4, 2]$ player P_1 is a dictator so P_2 and P_3 are dummies. In $[14 : 7, 7, 4]$ there are no dictators but both P_1 and P_2 have veto power while P_3 is a dummy. Finally, in $[10 : 5, 4, 2]$ all three players have veto power; there are no dictators or dummies.
3. In the weighted voting system $[q : 12, 7, 5, 1, 1]$ what is the smallest possible value of the quota q ? (14) what is the smallest quota which makes players P_4 and P_5 (22) dummies? What is the smallest quota which gives P_1 veto power? (15)
4. In the weighted voting system $[15 : 12, 8, 4, 2]$ what is the minimum percentage of votes needed to pass a motion? ($\frac{15}{26} = .5769$ so you need at least 58 percent of the vote) what should the quota be to require 75 percent approval for a motion to pass? (20)
5. In $[35 : 20, 15, 10, 5]$ what is the weight of the coalition $\{P_2, P_3, P_4\}$? (30) which players are critical in the coalition $\{P_1, P_2, P_3, P_4\}$? (Only P_1) what are the winning coalitions and what are their weights? $\{P_1, P_2, P_3, P_4\}(50)$, $\{P_1, P_2, P_3\}(45)$, $\{P_1, P_2, P_4\}(40)$, $\{P_1, P_3, P_4\}(35)$, and $\{P_1, P_2\}(35)$ How many times is each player critical? $B_1 = 5, B_2 = 3, B_3 = 1, B_4 = 1$. what is the Banzhaf power distribution of the system? $\beta_1 = 50\%$, $\beta_2 = 30\%$, $\beta_3 = 10\%$, $\beta_4 = 10\%$.
6. In the generic weighted voting system $[q : w_1, w_2, \dots, w_N]$ what is mathematically equivalent to the statement that P_1 is a dictator? ($w_1 \geq q$) what condition is mathematically equivalent to the statement that P_1 has veto power? ($w_2 + \dots + w_N < q$ and $w_1 < q$)
7. In a certain weighted voting system there are 5 players and the winning coalitions are all coalitions with 3 or more players one of which is P_1 what are the winning coalitions? $\{P_1, P_2, P_3, P_4, P_5\}$, $\{P_1, P_2, P_3, P_4\}$, $\{P_1, P_2, P_3, P_5\}$, $\{P_1, P_2, P_4, P_5\}$, $\{P_1, P_3, P_4, P_5\}$, $\{P_1, P_2, P_3\}$, $\{P_1, P_2, P_4\}$ $\{P_1, P_2, P_5\}$, $\{P_1, P_3, P_4\}$, $\{P_1, P_3, P_5\}$, $\{P_1, P_4, P_5\}$
 How many times is each player critical? $B_1 = 11, B_2 = 3, B_3 = 3, B_4 = 3, B_5 = 3$.
 what is the Banzhaf power distribution of the system? $\beta_1 = \frac{11}{23} = 48\%$, $\beta_2 = \beta_3 = \beta_4 = \beta_5 = \frac{3}{23} 13\%$.