Prep Exercises

1.1 Prep Exercise: Greatest Common Factor

Finding the GCF

Greatest common factor. Think about those three words. Greatest. Common. Factor. *What is the **greatest** factor **common** to all expressions?*

Example 1. Find the greatest common factor of 12 and 15.

Write each number in factored form:

12 = 2*2*3
15 = 3 * 5.

Take each factor common to both, however many times it's in common.

The greatest factor common to 12 and 15 is 3

The GCF is 3.

Example 2. Find the greatest common factor of 20 and 30.

Write each number in factored form:

20 = 2*2*5
30 = 2*5*3.

Take each factor common to both, however many times it's in common.

Both 20 and 30 contain the factors 2 and 5 (each one time),

The GCF is 2*5 = 10.

What operation holds two factors together?

Given one factor of 30, say 6, how do you find the other factor?

Example 3. Find the GCF of x^5 and x^3.

Write each expression in factored form:

$x^5 = x\,x\,x\,x\,x$

$x^3 = x\,x\,x$

The variable x is common to both **three** times,

$x^5 = x\,x\,x\,x\,x$

$x^3 = x\,x\,x$

The GCF = $x \cdot x \cdot x = x^3$

Checkpoint GCF 1

Compare your answers to the checkpoint problems with a colleague, then discuss any pattern you have found. Given two variable terms with the same variable but different powers, which of the two is the GCF, the lower power or the higher? Explain.

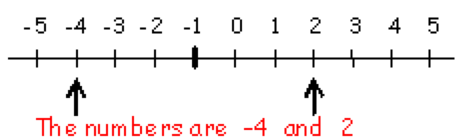
In symbols, suppose you are given $x^n + x^m$ and $n > m$, which is the GCF?

> When given any number of terms with the same variable to different powers, the GCF is the term with the **lowest** exponent.

1.3 Skill Prep Assignment 1 for Absolute Value….

For each of the following, find the numbers that have the given distance from each of the given numbers on the real number line.

Example 1. Find the number(s) that have a distance of 3 units away from the number -1 on the real number line.



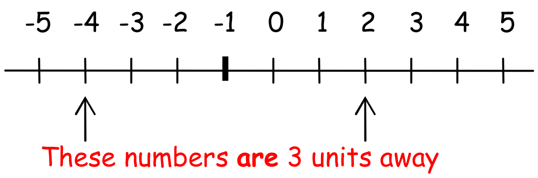
1. Find the number(s) that have a distance of 2 units away from the number 4.

2. Find the number(s) that have a distance of 5 units away from the number 0.

3. Find the number(s) that have a distance of 4 units away from the number -3.

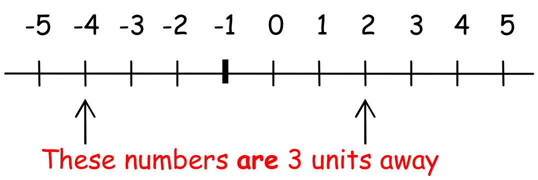
4. Find the number(s) that have a distance of 0 units away from the number 7.

Example 2. Find five numbers that have a distance **less** than 3 units away from the number -1 on the real number line.



There are many answers to this. One group of five numbers that is less than 3 units away is {-3, -2, 0, 1/2 and 2/3}

Example 3. Find five numbers that have a distance **more** than 3 units away from the number -1 on the real number line.



There are many answers to this. One group of five numbers that is more than 3 units away is {-6, -5, 3, 7/2 and 4.5}

5. Find five numbers that have a distance **less** than 2 units away from the number 4.

6. Find five numbers that have a distance **less** than 5 units away from the number 0.

7. Find five numbers that have a distance **less** than 4 units away from the number -3.

8. Find five numbers that have a distance **less** than 1 unit away from the number 0.

9. Find five numbers that have a distance **more** than 2 units away from the number 4.

10. Find five numbers that have a distance **more** than 1 unit away from the number 2.

1.3 Skill Prep Assignment 2 for Absolute Value

Factor out a 2 from each of the following expressions.

Example 1. $2x+6 = 2(x+3)$

Example 2. $2x-3 = 2(x-3/2)$

1. $2x-4$
2. $2x+10$
3. $2x-5$

Factor out a 3 from each of the following expressions.

Example 3. $3x-6 = 3(x-2)$

Example 4. $3x+2 = 3(x+2/3)$

4. $3x+9$
5. $3x-3$
6. $3x+4$

Factor out a 4 from each of the following expressions.

7. $4x-2$
8. $4x+8$
9. $4x-4$
10. $4x+3$

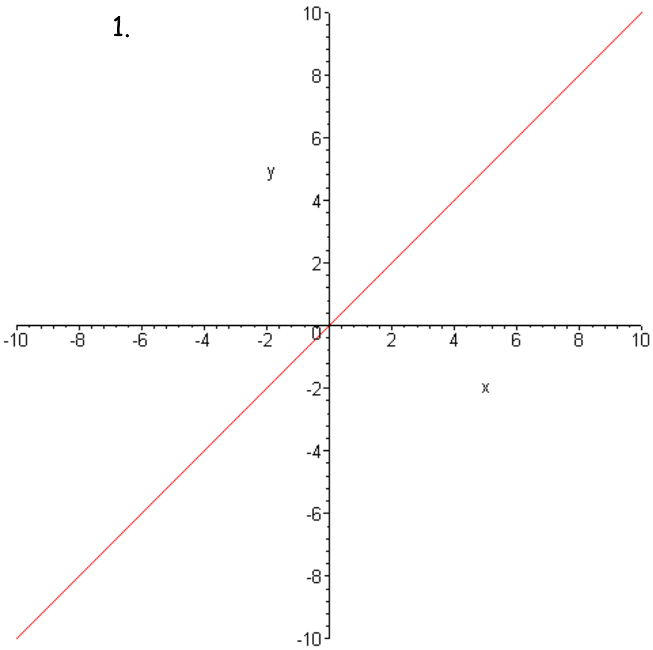
Challenge Problems:

1. Factor out a 2 from the expression
   $2x-\frac{1}{2}$
2. Factor out a 3 from the expression
   $3x+\frac{2}{3}$
3. Factor out a 1/3 from the expression
   $\frac{1}{3}x+3$
4. Factor out a 1/2 from the expression
   $\frac{1}{2}x+4$
5. Factor out a 2/3 from the expression
   $\frac{2}{3}x+\frac{1}{5}$

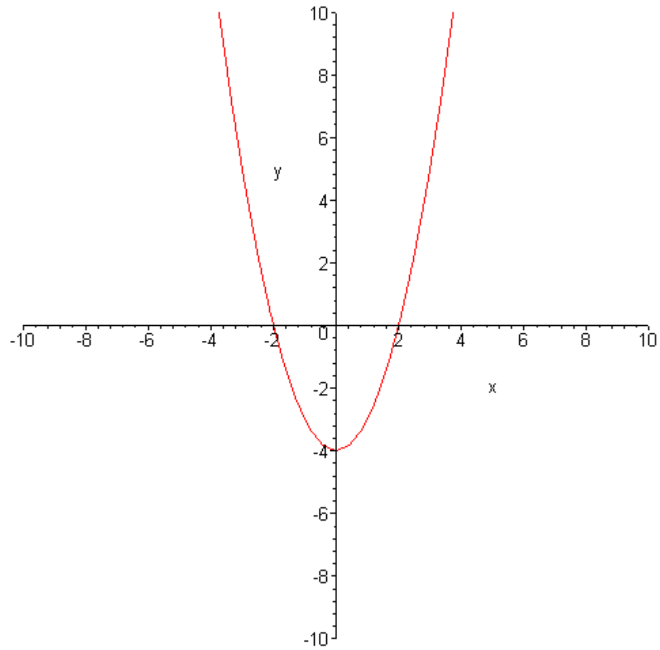
1.3 Prep Exercises for Absolute Value

Given the graphical representation of a function, sketch in the graph of the absolute value of the function.

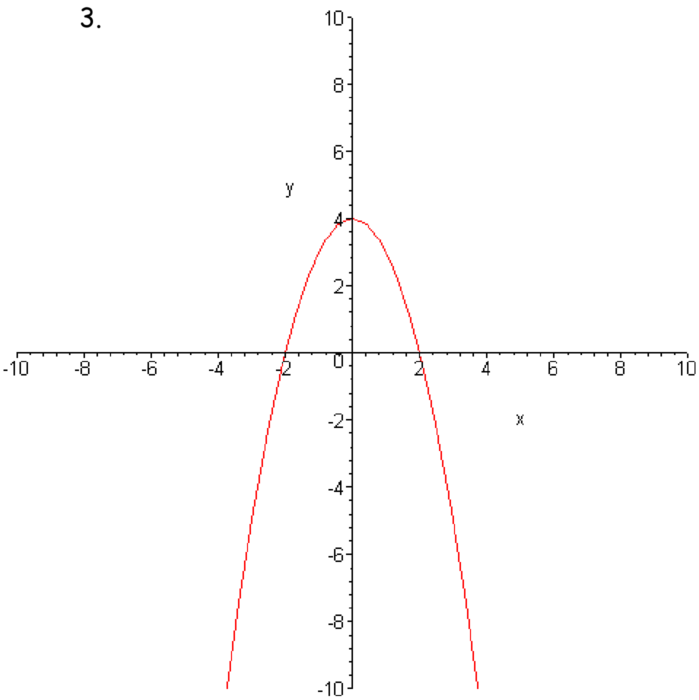
1.



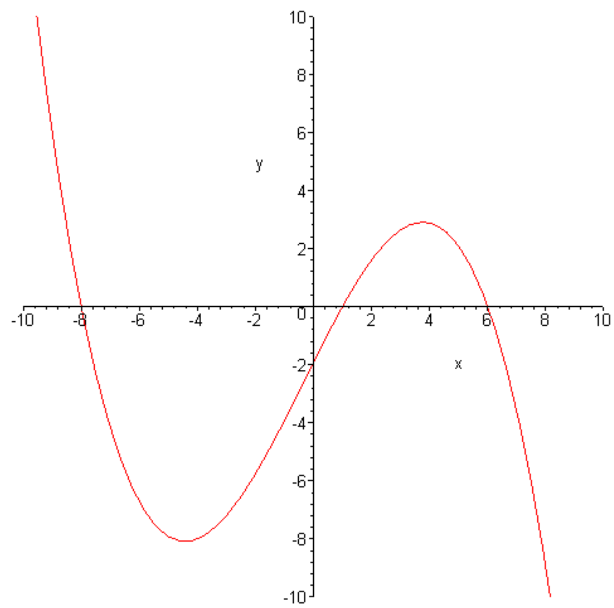
2.



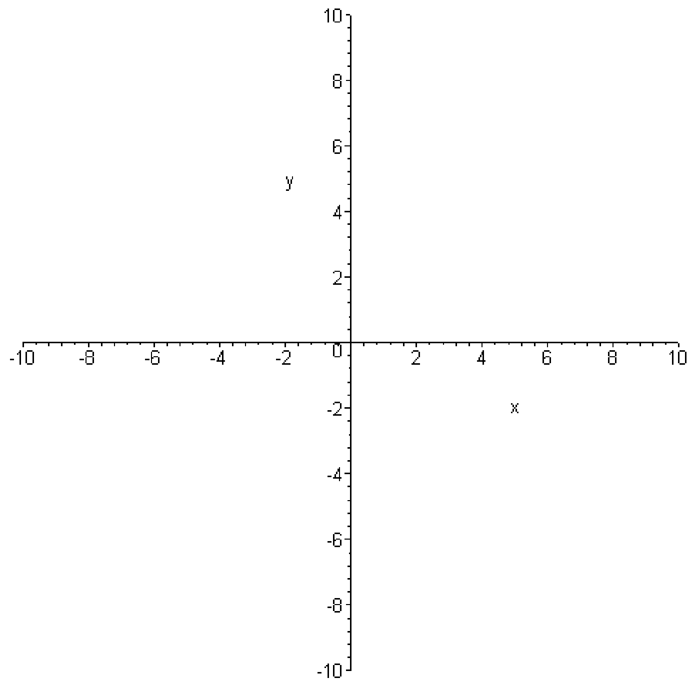
3.



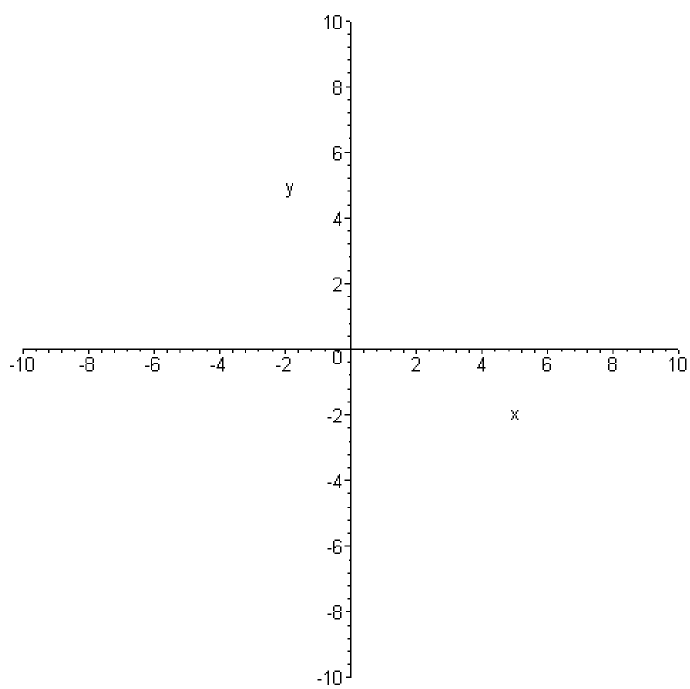
4.



5. Make a table of values and plot $y = |x|$ on the graph below.



6. Make a table of values and plot $y = |x^2 - 9|$ on the graph below.



1.4 Skill prep assignment
Part 1

Solve the following quadratic equations:

1. $x^2 + 11x = 28$
2. $x^2 - 4x = 12$
3. $x^2 = 9x - 20$
4. $x^2 - x = 3$
5. $5x^2 = 10x$
6. $2x^2 + x + 6 = 0$
7. $5x^2 + 12x + 7 = 0$
8. $6x^2 - 11x = 10$
9. $3x^2 + 3x + 1 = 0$
10. $4x^2 = 7x - 2$
11. $x^3 - x^2 = 2x$

Part 2

Another way to write a radical is as a rational (or fractional) exponent. For example, another way to write $\sqrt{(x+1)^3}$ is $(x+1)^{3/2}$. The numerator of the rational exponent is the exponent on the variable and the denominator of the rational exponent is the index of the radical. Notice, the whole quantity of x+1 is under the radical so there must be parentheses around this quantity. $(x+1)^{3/2}$ is not the same thing as $x+1^{3/2}$. In the second expression, only the 1 would be under the radical: $x + \sqrt{1^3}$

Example 1. $\sqrt{(x+1)^3}$ (index, exponent) is the same as $(x+1)^{3/2}$ (exponent ← index)

Remember, the index of a square root is 2.

Example 2. $\sqrt[3]{x-2}$ (index, exponent) is the same as $(x-2)^{1/3}$ (exponent ← index)

Remember, when you don't see an exponent, the exponent is a 1.

NOTE: **Don't forget that the exponent outside the parentheses CANNOT be distributed through:** $(x-2)^{1/3} \neq x^{1/3} - 2^{1/3}$

A. Switch the following radicals into rational exponents.

1. $\sqrt[5]{(x-3)^2} =$

2. $\sqrt[3]{(x+1)^4} =$

3. $\sqrt{(m+3)^7} =$

4. $\sqrt[4]{(3a-2)^5} =$

5. $\sqrt[7]{x+4} =$

6. $\sqrt{2n+5} =$

B. Switch the following rational exponents into radicals.

Example 3. exponent ↓ $(x+1)^{2/3}$ ← index is the same as index ↓ $\sqrt[3]{(x+1)^2}$ ↙ exponent

7. $(x-4)^{5/3} =$

8. $(m+3)^{1/4} =$

9. $(x+5)^{5/2} =$

10. $(4a-1)^{2/3} =$

11. $(2x+3)^{1/7} =$

12. $(5m-1)^{1/2} =$

1.4 Concept Prep Assignment

1. A soccer ball manufacturer knows that soccer balls are supposed to have a radius of 4.5 inches and be perfectly spherical in shape. He recently received an order for 100 balls. How much leather does he need to complete the order?

He knows that the formula for radius of a sphere is $r = \sqrt{\frac{S}{4\pi}}$ *, where S is the surface area.*

2. The function $S = \sqrt{30d}$ describes the relationship between *S*, the speed of a car in miles per hour, and *d*, the distance in feet a car skids after applying the brakes on a dry tar road (Kime and Clark, p. 219). If you were traveling 50 miles per hour, how long would your skid mark be?

3. I'm thinking of a number, x, whose square root is 3. What is the number?

Written symbolically, the equation looks like this: $\sqrt{x}=3$.
How did you find the answer? What operation will "undo" the square root operation?

Solve each of the following.

a. $\sqrt{x}=3$

b. $\sqrt{x}=4$

c. $-3+\sqrt{x}=5$

d. $2+\sqrt{x}=10$

e. $7-\sqrt{x}=4$

f. $8+\sqrt{x}=3$

g. $\sqrt[3]{x}=2$

h. $\sqrt[3]{x}+5=12$

i. $\sqrt[4]{x}-3=2$

j. $\sqrt[4]{x}+3=5$

4. Are the following pairs of equations (#1-3) equivalent (have the same solution)? Why or why not?

a. $x=3$; $x^2=9$ b. $x=5$; $x^2=25$ c. $x=5$; $5x=25$

d. Make a table of values then plot by hand the graph of $y=\sqrt{x}$ on graph paper.

e. Use your graphing calculator to graph both sides of equation #3a above. Let $Y1=\sqrt{x}$ and $Y2=3$

2.1 Prep Assignment for Exponential Growth

Try it #1

a) Suppose that when you graduate you are offered a job with the ABC Company with a starting salary of $40,000 and a 3% raise every year. What would your salary be after 3 years? Make a table of values and then plot these values on a coordinate plane. Let t = number of years since the beginning of the contract be your input and the salary be your output.

b) If you had a starting salary of $40,000 with annual raises of $1200, what would be your salary after 3 years? Make a table of values and then plot these values on a coordinate plane. Let t = number of years since the beginning of the contract be your input and the salary be your output.

Try It #2

Suppose that over Christmas break you have two job offers. Team A offers you $1,000,000 for 30 days work to be their sports consultant for their cross country ski team.

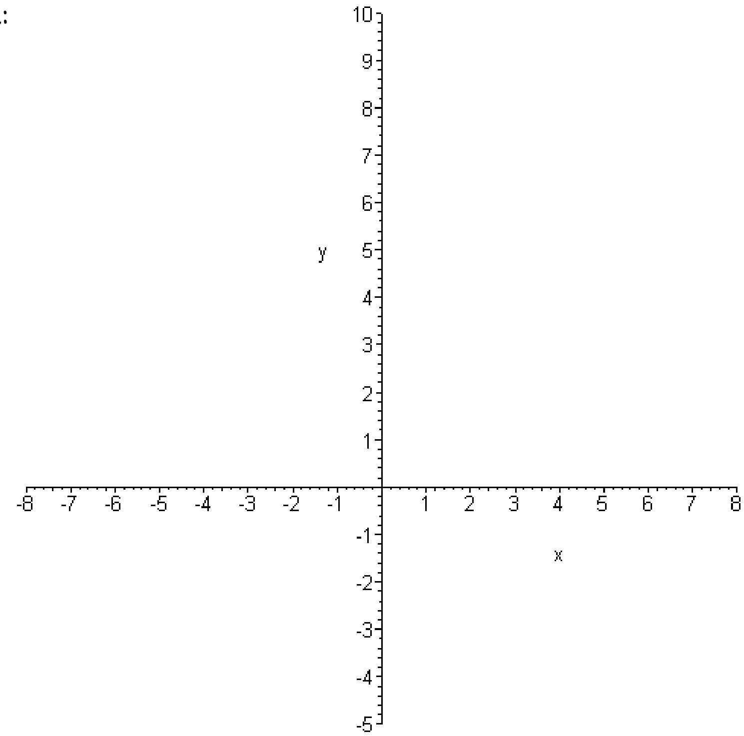
For the same work, Team B offers you $.01 for the first day of work, but will double that amount on the second day, double the new amount on the third day, and continue doubling every day for one month. They will give you the final doubled amount on day 30. Which job would you take and why?

2.3 Concept Prep Assignment: Graphs of Exponential Functions

Make a table of values of inputs and outputs for the exponential function with an initial value of 1 and a growth factor of 2 $\left(y = 2^x\right)$. Be sure to include both positive and negative values for inputs. Choose at least 8 values as inputs.

x	y

Make a sketch of your graph here:



Repeat the above for $y = 3^x$ and $y = 4^x$ then answer the following questions.

a. Describe, in words, the characteristics of the graphs.
b. How are all three graphs similar?
c. How are they different from each other?
d. What are possible values for the input ?
e. Complete: as $x \to \infty$, $y \to$???
   as $x \to -\infty$, $y \to$???
f. Will the output ever be 0? Why or why not?
g. Will the output ever be negative? Why or why not?
h. For what values of x is $2^x < 3^x$?
i. For what values of x is $2^x > 3^x$?
j. Write a formula for an exponential function whose graph hugs the y-axis more closely, i.e. is narrower than the graphs you sketched above.
k. Predict where the graphs of $y = 2.7^x$ and $y = 5^x$ would be located on your graph then test your prediction on your graphing calculator.

3.1 Concept Prep Assignment: Introduction to Logarithms

Try It!

With your small group, using your graphing calculators:

1. Go to TBLSET (2[ND] WINDOW). Let TblStart = 1; Δ Tbl = 1; Indpnt: Ask; and Depend: Auto.

2. Go to the **Y=** menu and in Y1= type in "log(x)."

3. Go to the TABLE menu (2[nd] GRAPH) and type in arbitrary values for *x*. The values for *log(x)* will appear in the output column (Y1).

4. Study the table and try to figure out what the "log" function means. Make a list of conjectures on your own paper, noting any patterns and relationships you find. (You might want to write your table on paper, too, since this feature of the calculator will only let you look at 7 rows at a time.)

Class Discussion

5. Now return to the TBLSET (2[ND] WINDOW) menu and Let TblStart = 0; Δ Tbl = 100; Indpnt: Auto; and Depend: Auto.

6. Study the new table and jot down any new relationships in the outputs that you see.

7. For what values is Y=log(*x*) undefined? Why do you think this is so?

8. In the table below, what is the relationship between the numbers in the columns?

1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

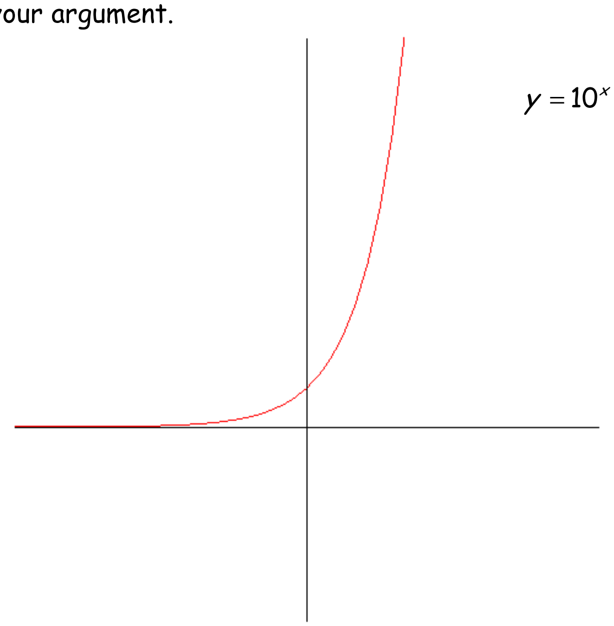
9. In the table below, what is the relationship between the numbers in the columns?

1	3
2	9
3	27
4	81
5	243
6	729
7	2187
8	6561
9	19683
10	59049

10 In the table below, what is the relationship between the numbers in the columns?

1	10
2	100
3	1000
4	10000
5	100000
6	1000000
7	10000000
8	1E+08
9	1E+09
10	1E+10

11. Discussion: Can any number be expressed as a power of 10 ? Use the graph of $y = 10^x$ below to support your argument.



3.3 Prep Assignment for the Natural Log Function

1. Use your calculator to evaluate the following:

   a) e^2
   b) e^4
   c) e^{-1}
   d) e^{-2}

2. Find the amount of money that will be available in 10 years in an account that pays 6.5% interest compounded continuously if you invest $5,000 now.

3. Which is the better deal: an account that pays 8% interest compounded monthly or one that pays 7.5% interest compounded continuously?

4. Suppose the coyote population in the Cuyahoga Valley National park can be estimated by the formula:

$$N(t) = 85e^{.03t}$$

   where t is the number of years after the year 2005.

   a) How many coyotes were in the park in 2005?

   b) About how many coyotes will be in the park in 2015?

   c) What is the relative rate of growth of the coyote population? Express your answer as a percentage.

5. Metro Parks of Columbus and Franklin County has used sharpshooters since 1993 to manage the population of deer in Sharon Woods Park. In 2003, there were approximately 65 deer per square mile in the park. If we use a continuous decay model with a decay rate of 18.17% for this scenario, how many deer per square mile were in the park in 1993 ?

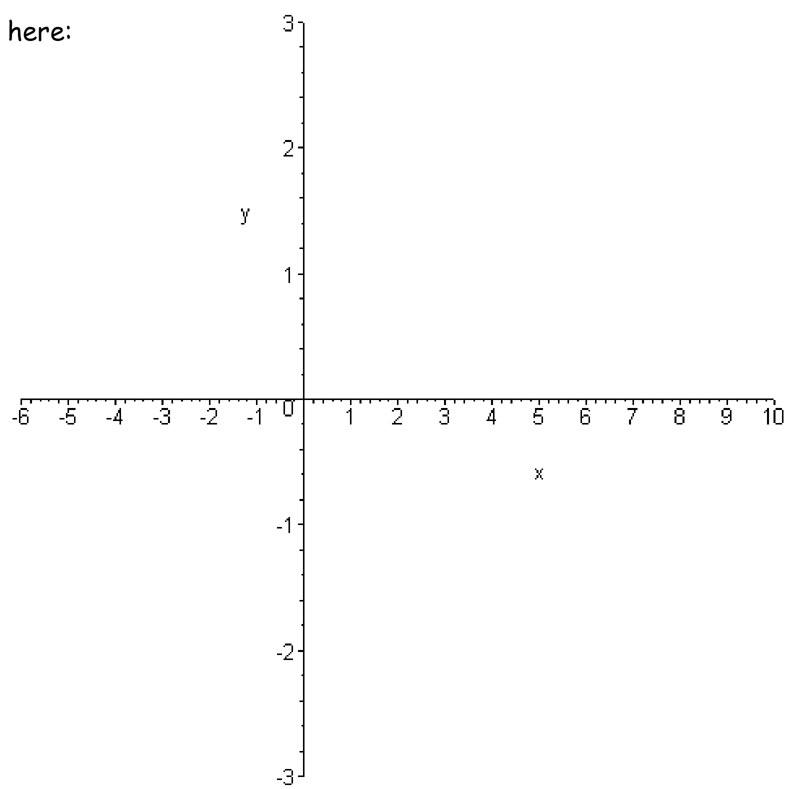
3.4 Prep Assignment for Graphs of Log Functions

Try it !

1. Complete the following table of values for the logarithmic function given by $y = \log x$. Be sure to choose at least 8 values for *x*, including a couple fractions between 0 and 1. (*Hint:* $\frac{1}{10}$ *and* $\frac{1}{100}$ *are good choices..*) What happens when you choose a negative value for the input? Try evaluating log(-10).

x	$y = \log x$

Make a sketch of the graph here:



2. Repeat the above for the functions given by $y = \log_2 x$ and $y = \log_4 x$.

3. Answer each of the following :

   l. Describe, in words, the characteristics of the graphs.
   m. How are all three graphs similar?
   n. How are they different from each other?
   o. How are these graphs related to the graphs of the exponential functions that you drew in section 3.4 ?
   p. What are possible values for the input ?
   q. Complete: as $x \to \infty$, $y \to$???

      as $x \to 0^+$, $y \to$???
   r. Can the input be 0? Why or why not?
   s. Can the input be negative? Why or why not?
   t. For what values of x is $\log x < \log_2 x$?
   u. For what values of x is $\log x > \log_2 x$?
   v. Write a formula for a logarithmic function whose graph hugs the x-axis more closely than the graphs you sketched above.

3.5 Prep Assignment: Solving Logarithmic Equations

1. Write each of the following in exponential form.

   a) $\log 100 = 2$ b) $\log_2 8 = 3$

   c) $\log_5 125 = 3$ d) $\ln e^2 = 2$

   e) $\log x = 3.2$ f) $\log(x-1) = 3$

   g) $\ln x = 5$ h) $\ln(x+1) = 7$

2. Write each of the following in logarithmic form.

   a) $10^x = 100$ b) $10^x = 324$

   c) $10^{x-1} = 100$ d) $10^{x+2} = 4321$

   e) $e^x = 5$ f) $e^{x-2} = 35$

   g) $e^x = 5$

4.1 Prep Assignment for Graphs of Rational Functions

1. Complete the following chart.

$x \rightarrow$	-10	-5	-2	-1	$-\frac{1}{2}$	$-\frac{1}{10}$	0	$\frac{1}{10}$	$\frac{1}{2}$	1	2	5	10
a) $f(x)=\frac{1}{x}$													
b) $g(x)=\frac{1}{x^2}$													
c) $h(x)=\frac{1}{x^3}$													
d) $f(x)=\frac{1}{x^4}$													

2. Complete the following chart:

	As $x \rightarrow \infty$, $y \rightarrow ??$	As $x \rightarrow -\infty$, $y \rightarrow ??$	As $x \rightarrow 0^+$ (as x gets close to 0 from the positive numbers), $y \rightarrow ??$	As $x \rightarrow 0^-$ (as x gets close to 0 from the negative numbers), $y \rightarrow ??$
a) $f(x)=\frac{1}{x}$				
b) $g(x)=\frac{1}{x^2}$				
c) $h(x)=\frac{1}{x^3}$				
d) $f(x)=\frac{1}{x^4}$				

3. Now graph each of the above functions, each on a separate graph. What do you notice about the graphs as $x \rightarrow \infty$, as $x \rightarrow -\infty$, as $x \rightarrow 0^+$, as $x \rightarrow 0^-$?

4.2 Prep Assignment

1. Complete the table of values below. Work down each column.

x	$f(x)=\frac{1}{x}$	$g(x)=\frac{1}{x}+3$	$h(x)=\frac{1}{x}+5$	$r(x)=\frac{1}{x}-1$
1	1			
2	$\frac{1}{2}$			
3	$\frac{1}{3}$			
5	$\frac{1}{5}$			
10	$\frac{1}{10}$			
20	$\frac{1}{20}$			
30	$\frac{1}{30}$			
50	$\frac{1}{50}$			
100	$\frac{1}{100}$			
1000	$\frac{1}{1000}$			
$x\to\infty$	$f(x)\to 0$	$g(x)\to ?$	$h(x)\to ?$	$r(x)\to ?$

2. Name the value(s) of x for which each of the following is undefined.

a) $f(x)=\frac{1}{x}$

b) $g(x)=\frac{4}{x^2}$

c) $y=\frac{3}{x}+5$

d) $m(x)=\frac{8}{x^2}-7$

e) $n(x)=\frac{7}{x-3}$

f) $y=\frac{2}{x+5}$

g) $h(x)=\frac{7}{x^2-1}$

h) $f(x)=\frac{10}{x^2-16}$

i) $y=\frac{7x}{x-3}$

j) $g(x)=\frac{10x}{x+15}$

k) $g(x)=\frac{x+4}{x-3}$

l) $g(x)=\frac{x-12}{x+5}$

m) $y=\frac{x-1}{x^2-2x-15}$

n) $y=\frac{x+5}{2x^2-9x-5}$

Prep Assignment 4.3A: Combining Fractions

Combine each of the following fractions using the indicated operation. Leave answers in improper form, NOT mixed number form $\left(\text{eg. } \frac{10}{3}, \text{ NOT } 3\frac{1}{3}\right)$.

1. $\frac{1}{3}+5$
2. $\frac{1}{2}+6$
3. $\frac{1}{8}-1$
4. $\frac{1}{10}-3$
5. $\frac{7}{18}+2$
6. $\frac{51}{100}+4$
7. $\frac{8}{3}-4$
8. $\frac{8}{5}+6$
9. $\frac{100}{101}-7$
10. $\frac{-20}{7}+9$

Prep Assignment 4.3B: Finding the LCD

Find the least common denominator (LCD) for each of the following pairs of numbers or expressions.

1. $\frac{1}{3}, 5$
2. $\frac{1}{2}, 6$
3. $\frac{1}{8}, \frac{1}{6}$
4. $\frac{1}{14}, \frac{3}{21}$
5. $\frac{1}{x}, \frac{5}{x^2}$
6. $\frac{8}{(x+3)}, \frac{7}{(x+3)^2}$
7. $\frac{6}{(x+3)}, 10$
8. $\frac{7}{(x+5)^3}, 30$
9. $\frac{7}{(x-3)^2}, 9$
10. $\frac{8}{(x+1)^2}, 11$