Checkpoint Exercises

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1.1 Checkpoint GCF 1

Name the GCF of the following:

1. $x^2 + x^9$

2. $x^{10} + x^2$

3. $x^{102} + x^2$

4. $6x^2y + 15ab^2 + 18$

5. $x^3y^2 - 5xy^3 + 7x^2y^2$

6. $4x^3y^2 - 8x^2y^3 + 2x^4y^4$

1.1. Checkpoint GCF 2

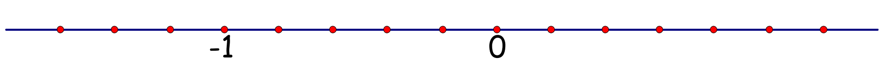
1. Circle the smaller number in each pair.

$-5, -10$ $\quad\quad$ $-8, -4$ $\quad\quad$ $-102, 1$

$\frac{1}{2}, \frac{1}{3}$ $\quad\quad$ $\frac{1}{4}, \frac{1}{101}$ $\quad\quad$ $\frac{2}{3}, \frac{2}{5}$

$-\frac{1}{6}, -\frac{1}{7}$ $\quad\quad$ $-\frac{1}{10}, -\frac{1}{20}$ $\quad\quad$ $-\frac{2}{7}, -\frac{2}{9}$

2. If you have a hard time comparing numbers, you might want to think of the number line. The smaller number is always to the ***left.*** Place the numbers $-\frac{1}{10}, -\frac{1}{5}$ on the number line below.



3. Complete the following.

WHAT DID THE PET OWNER SAY TO HER OVERWEIGHT PARROT?

Match the expression with its factor to be factored out. To answer the riddle, write the letter of the correct answers on the appropriate lines below.

1. $x^{-5} + x^{-4}$
2. $x^{5} + x^{-4}$
3. $x^{5} + x^{4}$
4. $x^{\frac{1}{5}} + x^{\frac{1}{4}}$
5. $x^{-\frac{1}{5}} + x^{-\frac{1}{4}}$
6. $x^{-6} + x^{-4}$
7. $x^{-3} + x^{4}$
8. $x^{-3} + x^{-7}$
9. $x^{\frac{1}{4}} + x^{\frac{1}{3}}$
10. $x^{-\frac{1}{4}} + x^{-\frac{1}{3}}$
11. $x^{3} + x^{4}$

Y. $x^{-\frac{1}{4}}$

P. x^{-5}

M. x^{-7}

A. $x^{-\frac{1}{3}}$

B. $x^{\frac{1}{3}}$

E. $x^{\frac{1}{4}}$

L. x^{4}

G. x^{5}

N. x^{-6}

O. x^{-4}

L. x^{3}

L. $x^{\frac{1}{5}}$

D. x^{6}

F. $x^{-\frac{1}{5}}$

O. x^{-3}

____ ____ ____ ____ ____ ------- ____ ____ ------- ____ ____ ____ ____

1 2 3 4 5 6 7 8 9 10 11

1.1. Checkpoint GCF 3

Factor the following. Then check by using the TABLE feature of your calculator AND the corresponding graphs.

1. $x^{-7} + x^{-14}$
2. $x^{-8} + x^{-14}$
3. $x^{5/2} + x^{11/2}$
4. $x^{9/2} + x^{19/2}$
5. $x^{9/2} + x^{13/2}$
6. $x^{-3/2} + x^{-13/2}$
7. $x^{-9/2} + x^{-13/2}$

1.2 Checkpoint Binomial GCF 1

Factor out the GCF of each of the following:

a) $8(x+2)^2 - 11x(x+2)$

b) $4(x+8)^2 + 3x(x+8)^3$

c) $(4x-3)(x-9)^2 + (4x-3)(x-9)$

1.2 Checkpoint Binomial GCF 2

Factor out the term with the lowest power:

a) $9(x+6)^{-7}+8x(x+6)^{-8}$

b) $5(x-4)^{-3}+9x(x-4)^{-10}$

c) $10x(x+2)^{-4}-15x(x+2)^{-5}$

1.2 Checkpoint Binomial GCF 3

Factor out the term with the lower power:

a) $5(x+4)^{\frac{9}{2}}+2x(x+4)^{\frac{7}{2}}$

b) $3(x-1)^{\frac{11}{2}}+2x(x-1)^{\frac{11}{4}}$

c) $3x(x+5)^{-\frac{5}{2}}+2x^2(x+5)^{\frac{7}{2}}$

1.2 Checkpoint Binomial GCF 4

Factor out the GCF:

a) $3x^2(7x-8)^7+49x^3(7x-8)^6$

b) $5x(4x+1)^3+75x^2(4x+1)^4$

c) $6x^5(3x-5)^8+15x^6(3x-5)^7$

1.3 Checkpoint Absolute Value 1

Make a number line to show possible values of x for each of the following situations. Then write your solution as an interval.

1. $|x| < 6$
2. $|x| \leq 6$
3. $|x| \geq 6$
4. $|x| > 8$

1.3 Checkpoint Absolute Value 2

Mark your possible positions on the number line for each of the equations or inequalities below. Then write your solution using interval notation, where appropriate.

1. $|x-3| = 2$
2. $|x-1| = 5$
3. $|x+4| = 1$
4. $|x+3| = 4$
5. $|x-3| < 2$
6. $|x-1| \leq 5$
7. $|x+4| < 1$
8. $|x-3| \geq 2$
9. $|x-1| > 5$
10. $|x-1| \geq 7$
11. $|x+3| = -4$
12. $|x-3| < -2$

Think about an algebraic way to solve the above and discuss it with your partner.

2.1 Checkpoint Exponential Functions 1

1. Write an exponential function with an initial value of \$50,000 and a growth rate of 8%.

2. Write an exponential function with an initial value of \$1,000 and a growth rate of 4%.

3. How much money would you have after 4 years if you invested \$3,000 in a Certificate of Deposit (CD) earning 5.5% interest compounded every year?

4. How much money would you have after 40 years if you invested \$1,000 in a mutual fund earning 8% interest compounded every year?

5. Explain **why** in the formula $A(t) = C \times 2^t$ the constant C represents the beginning amount of your quantity, given that the variable t represents time.

6. Are the following exponential functions? Yes or no and **why.** For those that are exponential functions, identify the initial amount and the growth rate.

   a) $y = 10(2)^n$

   b) $y = 5n^2$

   c) $y = (n)^{.5}$

   d) $y = .1(1.02)^n$

   e) $y = 300(1.75)^n$

   f) $y = n^5$

   g) $y = -5n + 1$

   h) $y = 10(2)^n$

7. For each of the following growth rates, name the growth factor.

   a) 5%

   b) 7.5%

   c) 42%

   d) 100%

   e) 200%

   f) 150%

8. For each of the following growth factors, name the growth rate.

   a) 1.06

   b) 1.15

   c) 1.50

   d) 5.00

   e) 2.00

   f) 3.5

2.1 Checkpoint Exponential Functions 2

1. Determine the amount of money you would have in 10 years if you invested $3,000 in an account that gives 6.5% interest compounded:

    a) annually
    b) semi-annually
    c) quarterly
    d) monthly
    e) daily

2. Determine the amount of money you would have in 10 years if you invested $ 3,000 in an account that compounds interest monthly if the interest rate was:

    a) 2.5 %
    b) 4 %
    c) 5.5 %
    d) 8 %
    e) 10.5 %

3. Suppose that you need $5,000 in 3 years and found an account that will give you 8 % interest compounded quarterly. How much would you need to invest now so that you will have that $5,000 in three years?

4. A one-page letter is folded into thirds to go into an envelope. If it were possible to repeat this kind of tri-fold 20 times, how many miles thick would the letter be? (A stack of 150 pieces of stationery is one inch thick: 1 mile = 5280 feet.)*

5. The population of a small town increases by a growth factor of 1.152 over a two-year period.

    a) By what percent does the town increase in size during the two-year period?
    b) If the town grows by the same percent each year, what is its annual percent growth rate? *

NOTE: Problems #4 and #5 are adapted from Connally, Hughes-Hallett, Gleason et al. Functions Modeling Change. A Preparation for Calculus. New York: John Wiley & Sons, 2000, p. 111.

2.2 Checkpoint Exponential Functions: Decay

1. Write an exponential function with an initial value of 50,000 and a decay rate of 3%.

2. Write an exponential function with an initial value of 1,000 and a decay rate of 5%.

3. How many people would live in a town after 5 years if 30,000 people lived there originally and 10% skipped town every year?

4. How many people would live in a town after 8 years if 10,000 people lived there originally and 1% skipped town every year?

5. Explain why in the formula $A(t) = C(2)^t$ that the constant C represents the beginning amount of your quantity, given that the variable t represents time.

6. The amount (in milligrams) of a drug in the body t hours after taking a pill is given by $A(t) = 25(0.85)^t$.

   a) What is the initial dose given?
   b) What percent of the drug leaves the body each hour?
   c) What is the amount of the drug left after 10 hours?
   d) After how many hours will there be less than 1 milligram left in the body?*

*NOTE: Problem #6 is from Connally, Hughes-Hallett, Gleason et al. Functions Modeling Change. A Preparation for Calculus. New York: John Wiley & Sons, 2000, p. 111.

2.2 Checkpoint Exponential Decay 2: Factors and Rates

1. Identify each of the following as a growth or decay exponential function. Identify the growth or decay factor, rate, and the initial value.

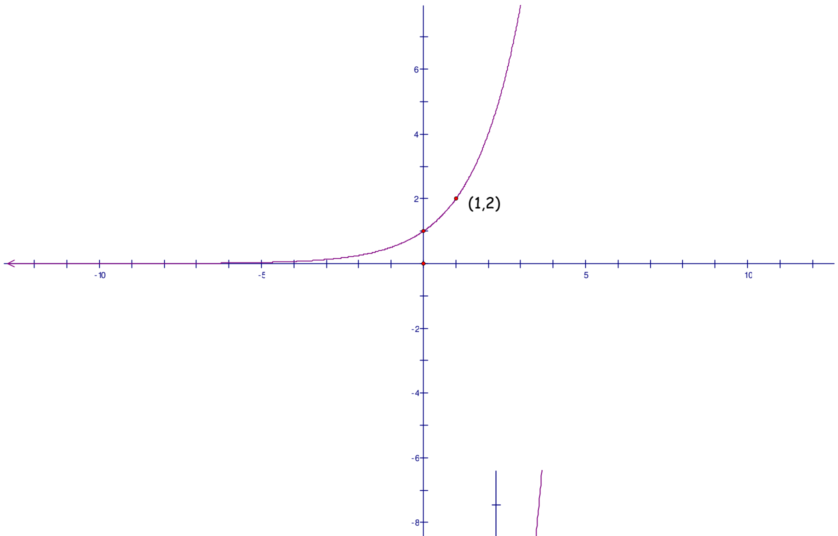
   a) $y = 10(1.02)^n$
   b) $y = 5\left(\frac{1}{3}\right)^n$
   c) $y = .5(4)^n$
   d) $y = .1(0.2)^n$
   e) $y = 300(.75)^n$
   f) $y = (5)^n$
   g) $y = 15(8)^{-n}$
   h) $y = -6(4)^n$

2. Create a real world scenario that could be modeled by each of the above exponential models.

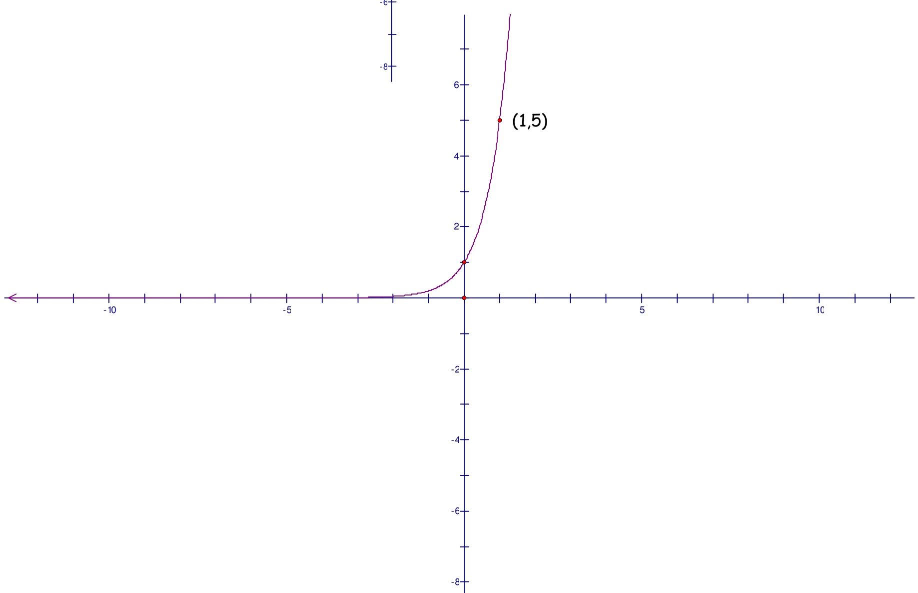
2.3 Checkpoint: Graphs of Exponential Functions

Write a function rule for each of the graphs below.

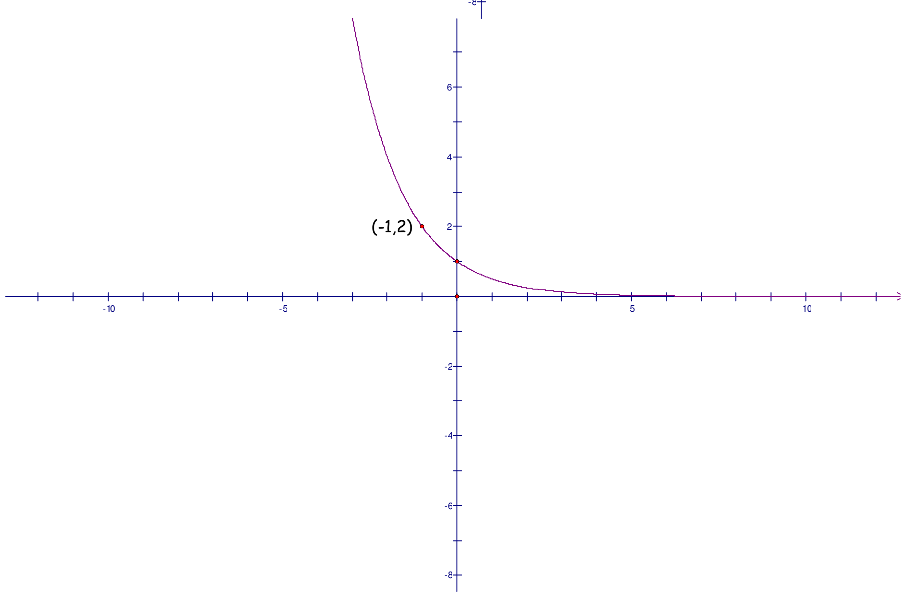
1.



2.



3.



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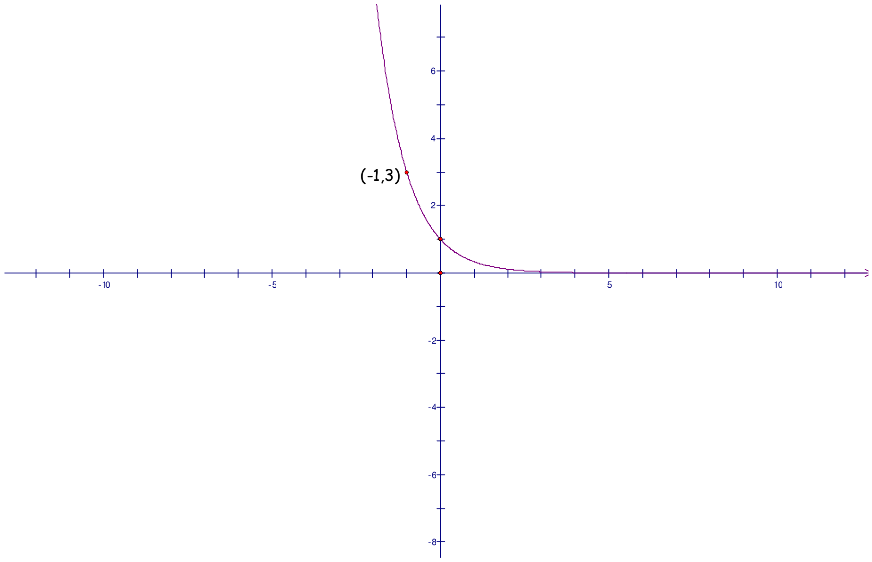
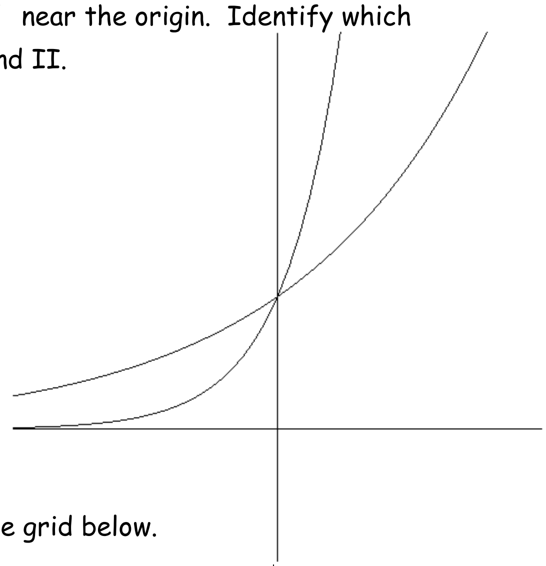


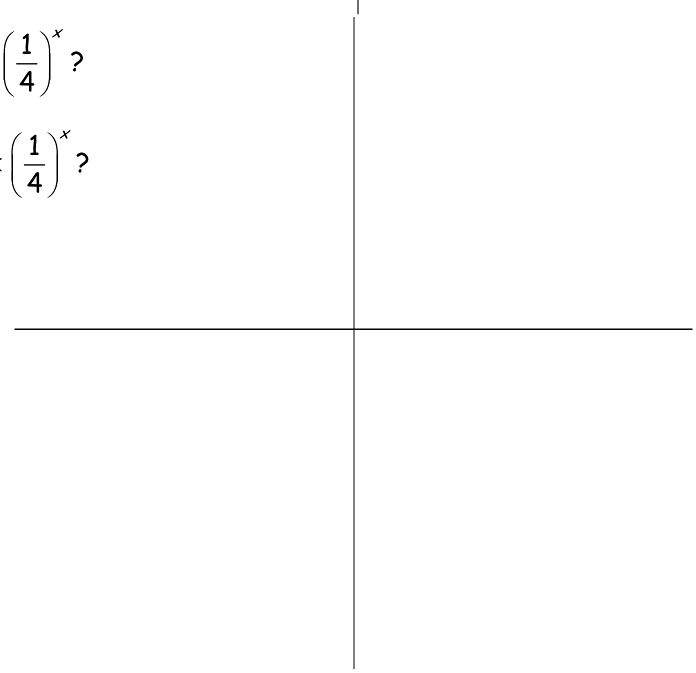
5. Below are the graphs of $y = 2^x$ and $y = 10^x$ near the origin. Identify which graph goes with each function in quadrants I and II.



6. Plot a graph of $y = \left(\frac{1}{3}\right)^x$ and $y = \left(\frac{1}{4}\right)^x$ on the grid below.

a. For what values of x is $\left(\frac{1}{3}\right)^x > \left(\frac{1}{4}\right)^x$?

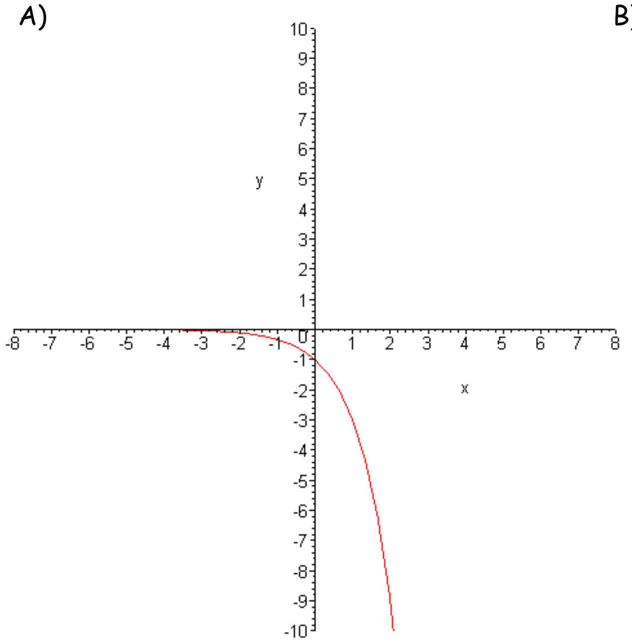
b. For what values of x is $\left(\frac{1}{3}\right)^x < \left(\frac{1}{4}\right)^x$?



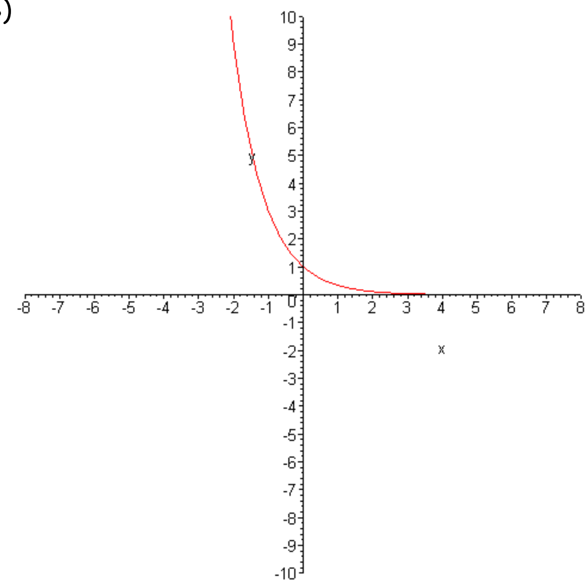
2.3 Checkpoint 2: Graphs of Exponential Functions

1. Without using your graphing calculator, match each of the following graphs with its function rule.

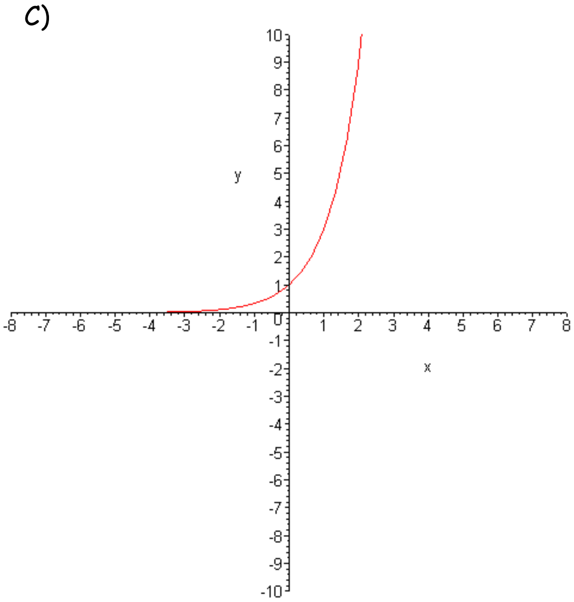
A)



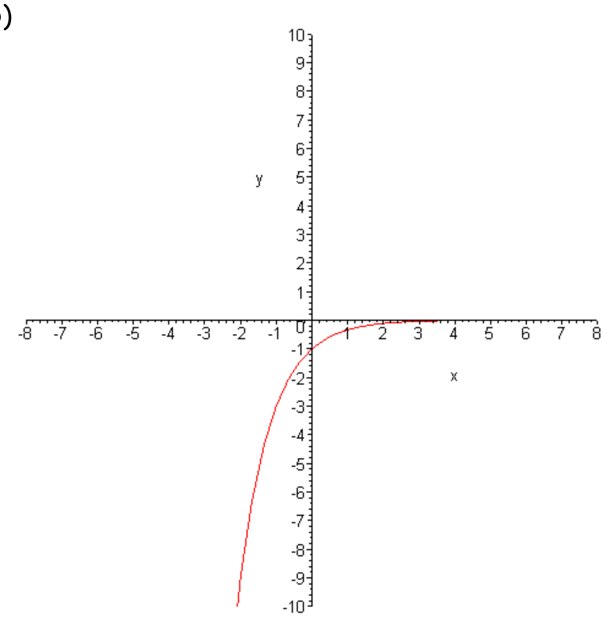
B)



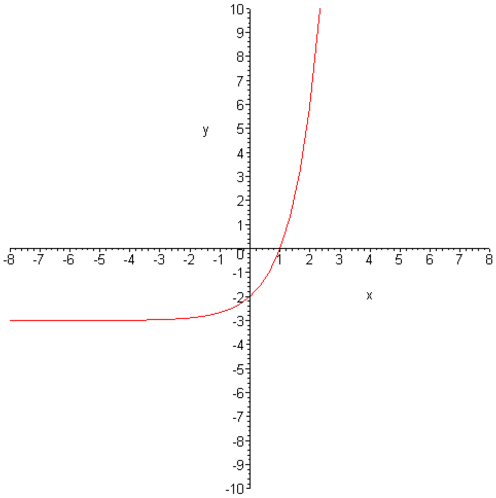
C)



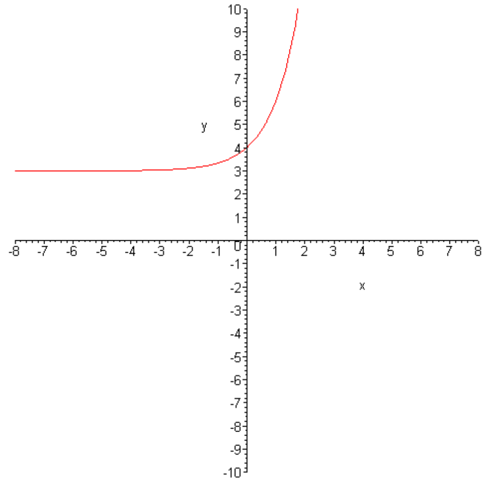
D)



E)



F)



i) $y = 3^x - 3$

ii) $y = -3^x$

iii) $y = 3^x$

iv) $y = 3^{-x}$

v) $y = 3^x + 3$

vi) $y = -3^{-x}$

2. Are the graphs of the following functions concave up or concave down? Explain

a) $y = x^2$

b) $y = -x^2$

2.4 Checkpoint: Continuous Compounding

1. How much money is an account after 8 years if you deposit $1500 in an account giving 5.5% interest compounded continuously?

2. How much money is an account after 2 years if you deposit $4500 in an account giving 4.5% interest compounded continuously?

3. How much money is an account after 40 years if you deposit $2000 in an account giving 6.5% interest compounded continuously?

4. How much money would you need to invest now in an account that gives 4% interest compounded continuously if you want to have $5000 in this account in 5 years?

5. It can be shown that $e = 1 + \frac{1}{1} + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \ldots$, where the approximation improves as more terms are included.

   a. Use your calculator to find the sum of the five terms above
   b. Find the sum of the first seven terms
   c. Compare your sums with the calculator's displayed value for e (which you can find be entering e^1) and state the number of correct digits in te five and seven term sum.
   d. How many terms of the sum are needed in order to give a nine decimal digit approximation equal to the calculator's displayed value for *e?* *

*NOTE: Problem #5 from Connally, Hughes-Hallett, Gleason et al. Functions Modeling Change. A Preparation for Calculus. New York: John Wiley & Sons, 2000, p. 178.

2.5 Checkpoint Exponential Equations

Solve and check.

1. $2^x = 2^{3x-1}$
2. $2^x = 4^{x+2}$
3. $2^{x+5} = 8^x$
4. $9^x = 27^{x-4}$
5. $4^{x+1} = 2^{3x-1}$

3.1. Checkpoint: Introduction to Logarithms

Without using your calculator, name the WHOLE NUMBER PART (the number to the left of the decimal) of the common logarithm of the following numbers.

1. 623
2. 49,456,900
3. 93 000 000
4. 0.001
5. 4.03

3.2 Checkpoint Logarithms

Find the indicated logarithms.

1. $\log(10000)$
2. $\log(0.01)$
3. $\log(0.001)$
4. $\log(100000)$
5. $\log_2(8)$
6. $\log_2(64)$
7. $\log_2\left(\frac{1}{4}\right)$
8. $\log_2\left(\frac{1}{16}\right)$
9. $\log_3(27)$
10. $\log_5(125)$
11. $\log_5\left(\frac{1}{25}\right)$
12. $\log_4(64)$
13. $\log_4(4^6)$
14. $\log(1)$
15. $\log_4(1)$
16. $\log_3(1)$
17. $\log_5(1)$
18. $\log_5(5^{-3})$
19. $\log_5(5^{102})$
20. $\log(10^{1.3})$
21. $\log_9(3)$
22. $\log_{25}(5)$
23. $\log_{25}(125)$
24. $\log_9(27)$
25. $\log_{16}\left(\frac{1}{64}\right)$
26. $\log_{25}\left(\frac{1}{125}\right)$
27. $10^{\log(100)}$
28. $2^{\log_2(8)}$
29. $10^{\log(1000)}$
30. $4^{\log_4(16)}$

3.3 Checkpoint: The Natural Log Function

1. Express each equation in exponential form.

   a) $\ln 5 = y$ b) $\ln x = 10$

   c) $\ln(x-3) = 4$ d) $\ln(x+5) = 32$

2. Express each equation in logarithmic form.

   a) $e^x = 102$ b) $e^5 = t$

   c) $e^{x+2} = 10$ d) $e^{0.5} = y$

3. Evaluate each of the following without the use of your calculator.

   a) $\ln 1$ b) $\ln e^5$

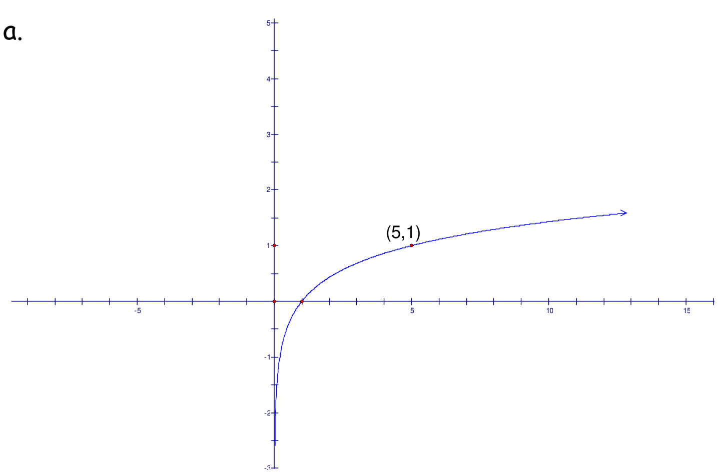
   c) $\ln e^{-4}$ d) $\ln \frac{1}{e}$

3.4 Checkpoint: Graphs of logarithmic functions

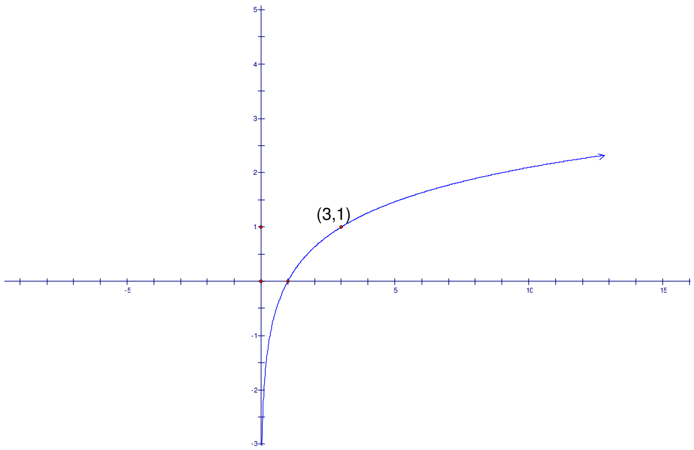
1. Identify the characteristics that graphs of all logarithmic functions of the form $y = \log_b x$ share.

2. Write a function rule for each of the graphs below.

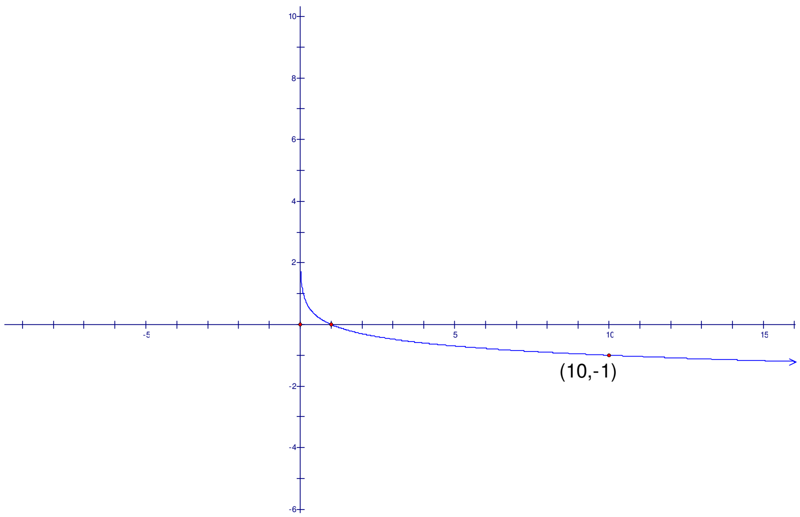
   a.



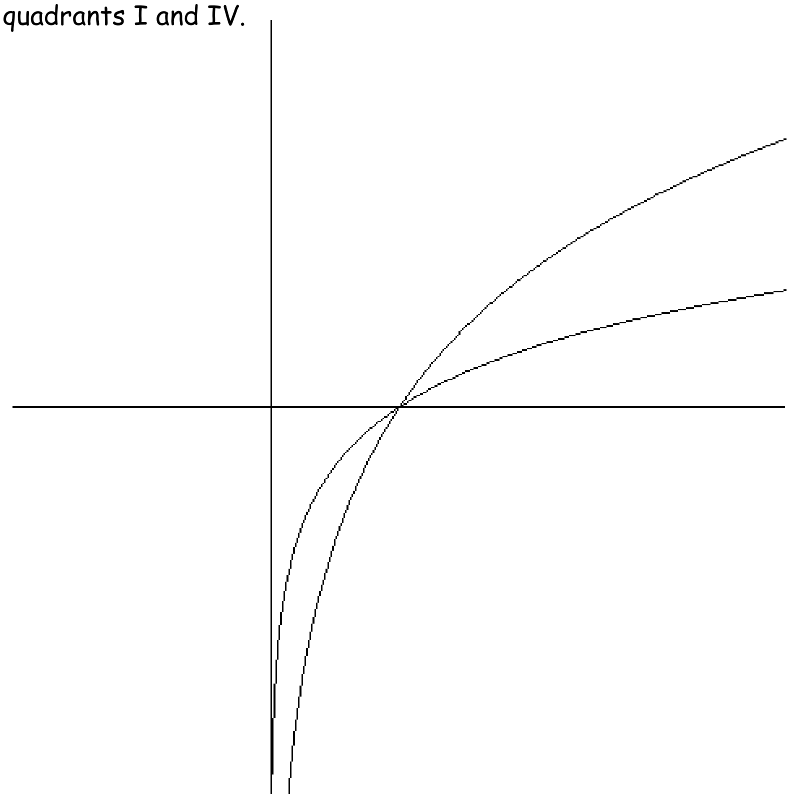
b.



c.



3. Below are the graphs of $y = \log x$ and $y = \ln x$ near the origin. Identify which graph goes with which function rule in quadrants I and IV.



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3.4 Checkpoint: Graphs of logarithmic functions 2

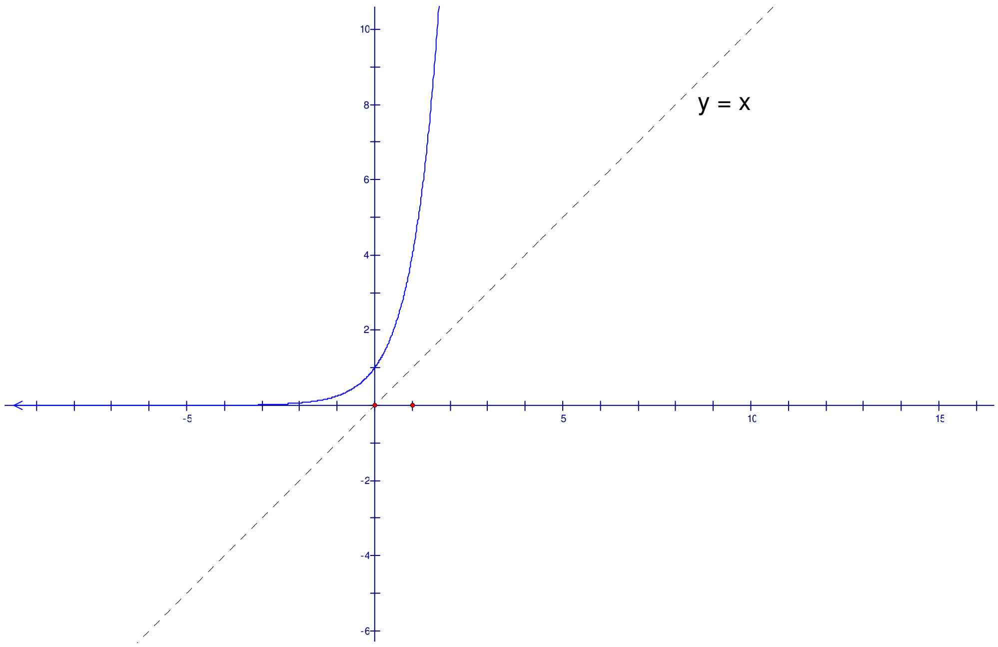
1. For each number x below, evaluate 10^x. Then take the common log of your answer.

x	10^x	$\log 10^x$
3		
4		
5		
-3		
-4		

2. For each number x below, evaluate $\log x$. Then use your answer as an exponent with a base of 10.

x	log x	$10^{\log x}$
10		
100		
1000		
$\frac{1}{10}$		
$\frac{1}{100}$		
$\frac{1}{1000}$		

3. Given the graph below of $y = b^x$, sketch in the graph of $y = \log_b x$.



3.4 Checkpoint: Domains of Logarithmic Functions

Find the domain of each of the following functions.

1. $y = \log(x + 12)$
2. $y = \ln(4x + 2)$
3. $y = \log(x^2 - 16)$
4. $y = \ln(x^2 - 100)$
5. $y = \log(x^2)$

3.5 Checkpoint Logarithmic Equations

Solve and check.

1. $\log_8 x = 2$
2. $\log_{\frac{1}{5}} 25 = x$
3. $\log_{\frac{1}{3}} \frac{1}{27} = x$
4. $\log_x (9x - 20) = 2$
5. $\log_x (2x + 15) = 2$

Checkpoint 4.1A: Graphs of Simple Rational Functions

Write each of the following symbolic expressions in words:

1. As $x \to \infty, f(x) \to 5$
2. As $x \to -\infty, f(x) \to 3$
3. As $x \to \infty, f(x) \to -7$
4. As $x \to -\infty, f(x) \to -7$
5. As $x \to -\infty, f(x) \to 2$
6. As $x \to 3^+, f(x) \to \infty$
7. As $x \to 4^-, f(x) \to \infty$
8. As $x \to 4^-, f(x) \to -\infty$
9. As $x \to 1^-, f(x) \to \infty$
10. As $x \to 1^-, f(x) \to -\infty$

Now make a sketch of each of the above. For each expression, you will only have one piece of the graph of *f*, but there may be more than one possible way of sketching it.

Checkpoint 4.1B: Graphs of Reciprocal Functions

Identify the horizontal and vertical asymptotes for each of the following functions. Use proper symbols to verify your choices. The find the *x* and *y* intercepts if they exist.

1. $f(x) = \frac{1}{x^3}$
2. $g(x) = \frac{1}{x^5}$
3. $h(x) = \frac{1}{x^4}$
4. $f(x) = \frac{1}{x^6}$
5. $g(x) = -\frac{1}{x}$
6. $h(x) = -\frac{1}{x^2}$

Checkpoint 4.2A:

Make a table of values for $\left(x, f\left(x\right)\right)$ as $x \to \infty$ like in the text. Then do the same for $x \to -\infty$ and determine the horizontal asymptote for each of the functions represented below.

1. $f(x) = \frac{1}{x} + 5$

2. $f(x) = \frac{1}{x} + 6$

3. $f(x) = \frac{1}{x} - 1$

4. $f(x) = \frac{1}{x} - 3$

5. $f(x) = \frac{1}{x^2} + 2$

6. $f(x) = \frac{1}{x^2} + 4$

7. $f(x) = \frac{3}{x} - 4$

8. $f(x) = \frac{8}{x} + 6$

9. $f(x) = \frac{100}{x^2} - 7$

10. $f(x) = \frac{-20}{x^2} + 9$

Now make a conjecture (guess) about the horizontal asymptote of a reciprocal function of the form $f(x) = \frac{c}{x^n} + k$. Explain your reasoning.

Checkpoint 4.2B:

Determine the horizontal asymptote of the graph of each of the functions, f, below. Verify your answer.

1. $f(x) = \frac{3}{x} - 1$

2. $f(x) = \frac{1}{x} - 5$

3. $f(x) = -\frac{6}{x} + 101$

4. $f(x) = -\frac{8}{x} + 20$

5. $f(x) = \frac{1}{x^2} + 5$

6. $f(x) = \frac{1}{x^2} - 9$

7. $f(x) = -\frac{3}{x^2} + 102$

8. $f(x) = -\frac{1}{x^2} + 6$

9. $f(x) = \frac{100}{x^2} - 10$

10. $f(x) = \frac{-10}{x^2} + 8$

Checkpoint 4.2C:

Determine the vertical asymptote(s) of the graph of each functions indicated below.

1. $f(x)=\dfrac{1}{x}$

2. $g(x)=\dfrac{4}{x^2}$

3. $y=\dfrac{3}{x}+5$

4. $m(x)=\dfrac{8}{x^2}-7$

5. $n(x)=\dfrac{7}{x-3}$

6. $y=\dfrac{2}{x+5}$

8. $h(x)=\dfrac{7}{x^2-1}$

9. $f(x)=\dfrac{10}{x^2-16}$

10. $y=\dfrac{7x}{4x^2-11x-3}$

11. $g(x)=\dfrac{10x}{6x^2+x-2}$

12. $g(x)=\dfrac{x+4}{x^2+2x-24}$

13. $g(x)=\dfrac{x-12}{x+5}$

14. $y=\dfrac{x-1}{x^2-2x-15}$

15. $y=\dfrac{x+5}{2x^2-9x-5}$

Checkpoint 4.3A:

Combine terms in each of the function rules for f and express the function as a ratio of polynomials:

1. $f(x)=\dfrac{1}{x+5}-3$

2. $f(x)=\dfrac{1}{x-4}+7$

3. $f(x)=\dfrac{1}{x^2}+4$

4. $f(x)=\dfrac{1}{x^2}-9$

5. $f(x)=\dfrac{1}{(x+7)^2}-3$

6. $f(x)=\dfrac{1}{(x-4)^2}+2$

7. $f(x)=-3+\dfrac{1}{(x+1)^3}$

8. $f(x)=-2+\dfrac{1}{(x-3)^3}$

Checkpoint 4.3B:

Name the horizontal asymptote of the graph of each of the functions g represented below. Then find the x-intercept(s) and the y-intercept.

1. $g(x)=\dfrac{9x+2}{3x-5}$

2. $g(x)=\dfrac{6x-1}{5x-8}$

3. $g(x)=\dfrac{7x-5}{21x-3}$

4. $g(x)=\dfrac{4x+10}{5x-15}$

5. $g(x)=\dfrac{3x^2-5x-2}{x^2-2x-15}$

6. $g(x)=\dfrac{5x^2-7x+2}{x^2+x-12}$

7. $g(x)=\dfrac{6x^2-5x-6}{2x^2-x-1}$

8. $g(x)=\dfrac{10x^2+21x+9}{2x^2+3x-5}$