Syllabus for Math 21001 — Linear Algebra
December 2002 – Revised July 2003 for 5th Edition

The schedule below is based on three 50 minute classes per week. The core sections must be covered. Eigenvalues and eigenvectors can be covered with §3.4, with eigenspaces used as an example in §4.3, or this material can be covered as a single unit after vector spaces are introduced.

The schedule suggests a total of 35 classes for the core sections, plus 2 or 3 exams. There are 41 – 44 classes available. Any time remaining can be devoted to topics chosen by the instructor from the list of optional sections.

Students should be exposed to proofs in this course and should be expected to write some proofs in homework assignments. It is not necessary to include long complicated proofs of theorems in lecture.

TEXT: *Elementary Linear Algebra*, Larson, Edwards, and Falvo, **5th edition**.

CORE SECTIONS:

§1.1 (2 days) Systems of Linear Equations

basic definitions, parametric solutions, equivalent systems, elementary operations, free and pivot variables, echelon form, Gaussian elimination, consistent/inconsistent systems, possible numbers of solutions

§1.2 (2 days) Matrices and Gaussian Elimination

coefficient matrix, augmented matrix, row operations on matrices, row echelon form and reduced echelon form, implications for existence, uniqueness, and number of solutions

§§2.1/2.2 (3 days) Matrix Operations and Properties

addition, scalar and matrix multiplication, matrix multiplication in terms of linear combinations of columns or rows, matrix multiplication related to systems of equations, properties of matrix operations, zero matrix and identity matrix, transpose and its properties, proofs of basic properties of operations

§2.3 (2 days) Matrix Inverse

definitions, calculation using Gauss-Jordan elimination (put off proof of this method until section on elementary matrices), formula for 2×2 inverse, properties of inverses with proofs, inverses and systems of equations, cancellation

§2.4 (1 day) Elementary Matrices

elementary row and column operations via elementary matrices, proof (using elementary matrices) of Gauss-Jordan elimination method for finding inverses, (LU-factorization can be skipped)

§§3.1/3.2 (3 days) Determinants

minors, cofactors, definition of determinant in terms of cofactors, calculation by row or column expansion, shortcut for 3×3, effects of row operations with proofs, calculation via elementary operations and row or column reduction, basic properties with proofs

§3.3 (1 day) Properties of Determinants

determinant of transpose, scalar multiple, product, inverse, proofs of properties

§3.5 (2 days) Applications of Determinants

adjoint and its properties, inverse via adjoint, use for proof that $\det A \neq 0$ implies A^{-1} exists, Cramer's rule

CORE SECTIONS (continued):

§§4.1/4.2 (4 days) Vectors in $\mathbb{R}^n$, Vector Spaces

$\mathbb{R}^n$, vector space properties (axioms) for $\mathbb{R}^n$, geometric interpretation of vectors and operations in $\mathbb{R}^n$, axioms for general vector space (over $\mathbb{R}$), examples ($\mathbb{R}^n$, matrices, polynomials, continuous functions on $[a, b]$), basic properties of vector spaces with proofs, emphasize concrete examples instead of "abstract" vector spaces

§4.3 (2 days) Subspaces

definition, subspace criteria, examples (in general spaces, $\mathbb{R}^n$, matrices, polynomials), sums and intersections of subspaces, eigenspaces (if eigenvectors covered with §3.4)

§4.4 (3 days) Spanning and Linear Independence

Spanning: linear combinations, span of a set, determine if a vector is in the span, relate to existence of a solution to a system of linear equations and to echelon form, description of the span of a set in $\mathbb{R}^n$ in terms of conditions on coordinates, spanning sets in general spaces and in $\mathbb{R}^n$ (relate to echelon form), minimum size of a spanning set in $\mathbb{R}^n$

Linear Independence: definition and examples, relate to uniqueness of solution of a system of equations and to echelon form, maximum size of a linearly independent set in $\mathbb{R}^n$, properties of linearly independent sets and spanning sets with proofs

§4.5 (3 days) Basis and Dimension

definition of basis, examples (standard bases for basic spaces), unique expression property, prove all bases have the same number of vectors, definition of dimension, basis and dimension for subspaces

§4.6 (3 days) Rank and Systems of Equations

row space, column space, and null space of a matrix, bases and dimensions for all (relate to echelon form), row rank equals column rank, Rank Plus Nullity Theorem, applications to systems of equations

§§3.4/7.1 (4 days) Eigenvalues and Eigenvectors of Matrices

definitions, characteristic polynomial, calculations of eigenvalues and eigenvectors, properties with proofs, trace is sum and determinant is product of eigenvalues, eigenvalues of similar matrices, inverses, subspace properties, eigenspaces, Cayley-Hamilton Theorem

OPTIONAL SECTIONS:

§§1.3/2.5 Applications

polynomial curve fitting, least squares regression, or pick from other applications

§§5.1/5.2/5.3 Inner Products

dot product, length and angles in $\mathbb{R}^n$, Cauchy-Schwarz inequality, inner product spaces, orthogonal projections, orthonormal bases (skip Gram-Schmidt)

§§6.1/6.2/6.3 Linear Transformations

matrix multiplication as a linear transformation, properties of linear transformations, eigenvalues and eigenvectors, coordinates (see also §4.7) and the matrix of a linear transformation, composition of linear transformations and relation to matrix multiplication, kernel and range