ZERO SETS OF POLYNOMIALS IN SEVERAL VARIABLES

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The following statement is well known, see e.g. [R].

Lemma 1. *Given* $k, n \in \mathbb{N}$

obvious that a system of ordered partitions of rank k for $\{1, \ldots, 2^n\}$ also works for $\{1, \ldots, l\}.$

Theorem 4. *Let $n \in \mathbb{N}$, n be odd, and let $Q(x)$ be n-homogeneous polynomial on $\mathbb{R}^N$. Provided $N > k!(\log_2(N))^k \binom{k+n-1}{k-1}$, there exists a linear subspace $X \hookrightarrow \mathbb{R}^N$, $\dim X = k$ such that $Q \equiv 0$ on X.*

PROOF. Write $Q = Q_1 + Q_3 + ¢¢¢ + Q_{2m+1}$

From the asymptotical point of view, it follows from the prime number theorem (see, e.g., [Ru, p.230]) that $\lim_{x \to \infty} \frac{\pi(2x)}{2\pi(x)} = 1$, so certainly $p(N) > \frac{N}{2}$ for N large enough. Thus for n large enough it suffices to choose $N \geq 4n$ in Proposition 5. On the other hand, it i2401(nothe)n[lhs