A	B	C	D	E	F	G	M	Σ

NAME...

Mathematics 11012 Intuitive Calculus A
Exam 3 April 22, 2010 R. M. Aron

Directions: Please answer questions A, B, C, and D in the space provided. ***Please write extremely neatly.*** **The rest of the Examination questions are to be answered on the "scan-tron" papers,** ***but you must show your work on this paper—even for the "scan-tron" part. Good luck!***

A.(a). Express the following as a power of e : $\frac{e^7e^{-4}}{e^1e^{-2}}$. $= \frac{e^3}{e^{-1}} = e^4$

(b). Simplify the following expression: $\ln e^5 + \ln(1/e)$. $= 5\ln e + \ln 1 - \ln e$

$= 5 + 0 - 1 = 4$

B. Find the derivative of each of the following functions:

(a). $f(x) = \frac{e^{2x}}{x^2-x}$. $f'(x) = \frac{(x^2-x)(2e^{2x}) - (2x-1)e^{2x}}{(x^2-x)^2}$

(b). $g(t) = e^{\ln(t^6+1)}$. $g(t) = t^6+1$, so $g'(t) = 6t^5$

(c). $f(z) = \ln(e^z - e^{-z})$. $f'(z) = \frac{1}{e^z-e^{-z}}(e^z + e^{-z})$

C. You want buy a house, paying for it totally with cash, on April 22, 2015. If the house will cost \$100,000 and a bank is offering a certificate of deposit with interest at 8% per year, compounded quarterly, how much should you deposit today to be able to purchase the house in 5 years?

Let P = amount to deposit. In 5 years, you'll have \$ $P(1.02)^{20}$. So, solve for P: $P(1.02)^{20} = 100{,}000$, getting $P = \frac{100\,000}{(1.02)^{20}} = \$67{,}297.13$.

D. Calculate each of the following:

(a). $\int (e^{2x} - \frac{2}{x})dx.$ $= \frac{e^{2x}}{2} - 2\ln x + C$

(b). $\int_1^9 (x - \frac{1}{\sqrt{x}})dx.$ $= \int_1^9 [x - x^{-1/2}]\,dx = \frac{x^2}{2} - 2x^{1/2}\Big]_1^9$

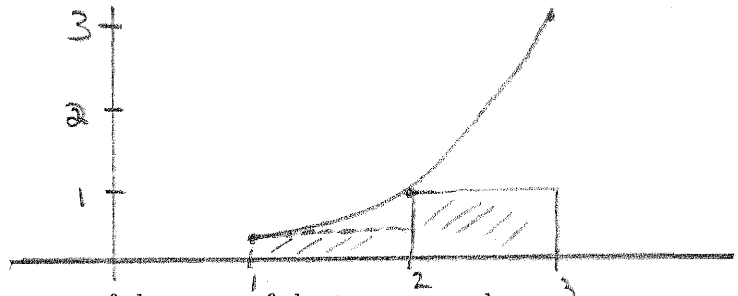
$= \left(\frac{81}{2} - 2\cdot 3\right) - \left(\frac{1}{2} - 2\right) = 40 - 4 = 36$

E. Let $f(x) = xe^{x^2} - 6.$

(a). Compute $f'(x) =$ $e^{x^2} + 2x^2 e^{x^2}$

(b). Compute $\int f'(x)dx =$ $xe^{x^2} + C.$

F. Consider the following diagram, consisting of two rectangles (as shown) between the x−axis and the curve $f(x) = x^3/9$, where x goes from 1 to 3:



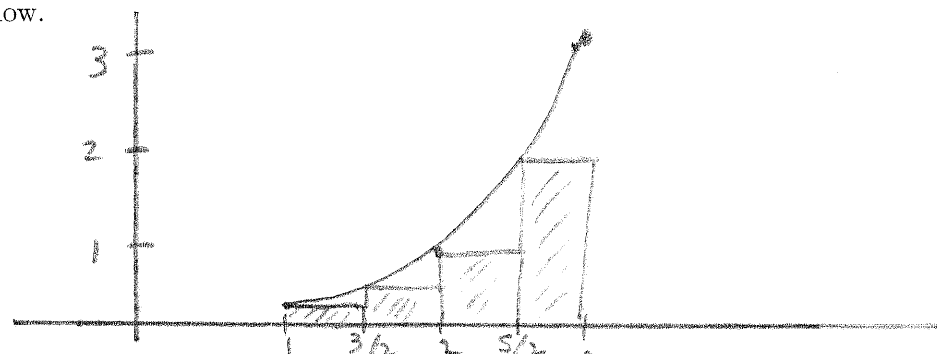
(a). Find the sum of the areas of the two rectangles.

$\frac{1}{9}\cdot 1 + \frac{8}{9}\cdot 1 = 1$

(b). Is the sum of the areas bigger or smaller than the area under the curve and above the x−axis?

Smaller

Now, subdivide each of the two rectangles, getting four rectangles, as drawn below.



(c). Find the sum of the areas of the four rectangles.

$\frac{1}{9}\cdot\frac{1}{2} + \frac{27}{72}\cdot\frac{1}{2} + \frac{8}{9}\cdot\frac{1}{2} + \frac{125}{72}\cdot\frac{1}{2} = 1.56$

(d). If this process were to be repeated indefinitely, what is the limiting value for the sums of the rectangles?

$\int_1^3 \frac{x^3}{9}\,dx = \frac{x^4}{36}\Big|_1^3 = \frac{81}{36} - \frac{1}{36} = \frac{80}{36} = \frac{20}{9} = 2.22$

G. The cost of maintaining a home generally increases as the home becomes older. Suppose that the rate of cost (dollars per year) for a home that is x years old is $200e^{0.5x}$. Find a formula for the total maintenance cost during the first x years. (Maintenance should be zero at $x = 0$.)

Given $r(x) = 200e^{.5x}$. $m(x)$ = maintenance cost from new to x years.

$$m(x) = \int_0 r(x)\,dx = \int 200e^{.5x}\,dx$$

$$= 200\int e^{\frac{1}{2}x}\,dx = 200\,\frac{e^{\frac{1}{2}x}}{1/2} + C$$

$$= 400e^{\frac{1}{2}x} + C.$$

At year 0, $m(0) = 0$ (given). So,

$m(0) = 400e^0 + C = 0$, or $C = -400$.

So, $m(t) = 400e^{\frac{1}{2}x} - 400$.

The rest of this Examination is to be done using the "scan-tron" sheet. Please write your work on this paper.

1. $\int \frac{1}{\sqrt[4]{x^3}} dx =$

   (a). $\frac{4}{7}x^{\frac{7}{4}} + C.$
   (b). $\frac{-3}{4}x^{\frac{-7}{4}} + C.$
   (c). $-3x^{\frac{-1}{3}} + C.$
   (d). $4x^{\frac{1}{4}} + C.$

$$\int \frac{1}{x^{3/4}}\, dx = \int x^{-3/4}\, dx$$
$$= 4x^{1/4} + C$$

2. The area under the curve $f(x) = x^2$ and above the $x-$axis, from $x = -1$ to $x = 1$, is:

   (a). 4 square units.
   (b). 2 square units.
   (c). 2/3 square units.
   (d). 0 square units.

$$\int_{-1}^{1} x^2\, dx = \frac{x^3}{3}\Big|_{-1}^{1} = \frac{1}{3} - \left(-\frac{1}{3}\right) = 2/3$$

3. $\int (x-1)(x+2)dx =$:

   (a). $(x^2/2 - 2x))(x^2/2 + 2x) + C.$
   (b). $2x + 1 + C.$
   (c). $x^3/3 + x^2/2 - 2x + C.$
   (d). $x^3/3 - 2x + C.$

$$\int (x^2 + x - 2)\, dx =$$
$$\frac{x^3}{3} + \frac{x^2}{2} - 2x + C$$

4. $\int_1^{e^2} \frac{3}{4t} dt =$:

   (a). 3/2.
   (b). 8/3.
   (c). $-1/4$.
   (d). 3/4.

$$\frac{3}{4}\int_1^{e^2} \frac{1}{t}\, dt = \frac{3}{4} \ln t \Big|_1^{e^2} = \frac{3}{4}\left(\ln e^2 - \ln 1\right)$$
$$= \frac{6}{4} = 3/2$$

5. Let $h(u) = (2u^5 - 3u + 6)^3$. Then $h'(u) =$:

   (a). $3(2u^5 - 3u + 6)^2.$
   (b). $3(10u^4 - 3)^2.$
   (c). $3(2u^5 - 3u + 6)^2(10u^4 - 3).$
   (d). $10u^5 - 3.$

$$3(2u^5 - 3u + 6)^2(10u^4 - 3)$$

6. You invest \$1,000 today, at 10% interest per year, with continuous compounding. In how many years will your investment triple to \$3,000?

(a). 10.99 years.

(b). 11.53 years.

(c). 10.62 years.

(d). 11.26 years.

Let x = # years.

$e^{.1x} = 3$, or $.1x = \ln 3$, or $x = \frac{\ln 3}{.1}$

$= 10.99$ years

7. What is the derivative of $f(x) = 3e^{x^2}$?

(a). $6e^{x^2}$.

(b). $6xe$.

(c). $6xe^{x^2}$.

(d). $3e^{x^2}$.

$3e^{x^2}(2x) = 6xe^{x^2}$

A	B	C	D	E	F	G	M	Σ

NAME..

Mathematics 11012 Intuitive Calculus B

Exam 3 April 22, 2010 R. M. Aron

Directions: Please answer questions A, B, C, and D in the space provided. ***Please write extremely neatly.*** **The rest of the Examination questions are to be answered on the "scan-tron" papers,** ***but you must show your work on this paper—even for the "scan-tron" part. Good luck!***

A.(a). Express the following as a power of e : $\frac{e^5 e^{-2}}{e^2 e^{-1}}$. $= \frac{e^3}{e^1} = e^2$,

(b). Simplify the following expression: $\ln e^4 - \ln(1/e)$. $= 4 \ln e - [\ln 1 - \ln e]$

$= 4 - [0 - 1] = 5$

B. Find the derivative of each of the following functions:

(a). $f(x) = \frac{e^x}{x^2-1}$. $f'(x) = \frac{(x^2-1)e^x - e^x(2x)}{(x^2-1)^2}$

(b). $g(t) = e^{\ln(t^4+t^2+1)} = t^4 + t^2 + 1$. So, $g'(t) = 4t^3 + 2t$

(c). $f(z) = \ln(e^z + e^{-z})$. $f'(z) = \frac{1}{e^z + e^{-z}} (e^z - e^{-z})$

C. You want buy a house, paying for it totally with cash, on April 22, 2015. If the house will cost \$100,000 and a bank is offering a certificate of deposit with interest at 4% per year, compounded continuously, how much should you deposit today to be able to purchase the house in 5 years?

Let P = amount to deposit. In 5 years, you'll have \$ $P(e^{.04})^5$. So, solve for P: $Pe^{.20} = 100,000$

or $P = \frac{100,000}{e^{.2}} = \$81,873.08$

D. Calculate each of the following:

(a). $\int(e^{3x} - \frac{3}{x})dx$. $= \frac{e^{3x}}{3} - 3\ln x + C$

(b). $\int_1^4(x + \frac{1}{\sqrt{x}})dx$. $= \int_1^4 (x + x^{-1/2})\,dx = \frac{x^2}{2} + 2x^{1/2}\Big]_1^4$

$= \left(\frac{4^2}{2} + 2\sqrt{4}\right) - \left(\frac{1}{2} + 2\right) = (8+4) - (\frac{1}{2} + 2)$

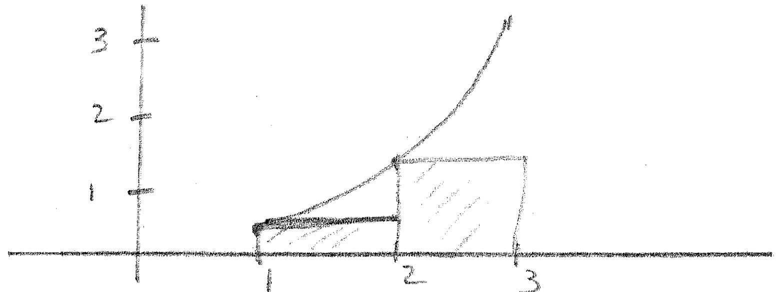
$= 9\frac{1}{2}$.

E. Let $f(x) = xe^{x^2} + 3$.

(a). Compute $f'(x)$. $= e^{x^2} + 2x^2 e^{x^2}$

(b). Compute $\int f'(x)dx$. $xe^{x^2} + C$

F. Consider the following diagram, consisting of two rectangles (as shown) between the $x-$axis and the curve $f(x) = x^2/3$, where x goes from 1 to 3.:



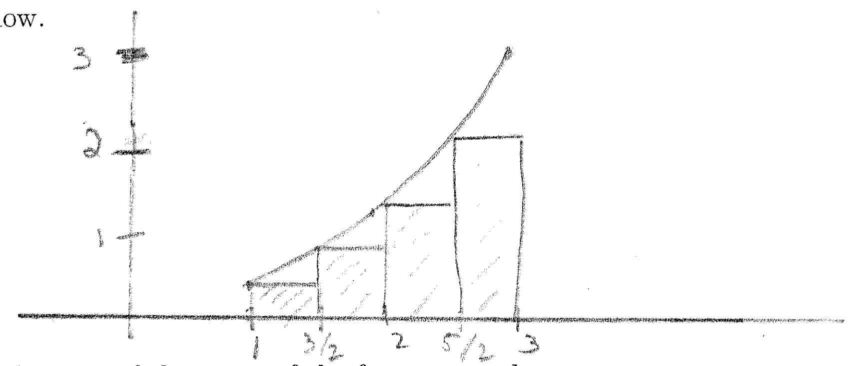
(a). Find the sum of the areas of the two rectangles.

$\frac{1}{3} \cdot 1 + \frac{4}{3} \cdot 1 = 5/3$

(b). Is the sum of the areas bigger or smaller than the area under the curve and above the $x-$axis?

Smaller

Now, subdivide each of the two rectangles, getting four rectangles, as drawn below.



(c). Find the sum of the areas of the four rectangles.

$\frac{1}{3} \cdot \frac{1}{2} + \frac{9}{12} \cdot \frac{1}{2} + \frac{4}{3} \cdot \frac{1}{2} + \frac{25}{12} \cdot \frac{1}{2} = 2.25$

(d). If this process were to be repeated indefinitely, what is the limiting value for the sums of the rectangles?

$\int_1^3 \frac{x^2}{3}\,dx = \frac{x^3}{9}\Big|_1^3 = \frac{27}{9} - \frac{1}{9} = \frac{26}{9} = 2.89$

G. An ice cube tray filled with tap water is placed in the freezer, and the temperature of the water is changing at the rate of $-12e^{-0.3t}$ degrees per hour after t hours. The original temperature of the tap water was 70 degrees. Find a formula for the temperature of water that has been in the freezer for t hours.

rate $r(t) = -12e^{-.3t}$, Temperature $T(t)$

$= \int r(t)\,dt = \int -12e^{-.3t}\,dt =$

$-12\int e^{-.3t}\,dt = -12\dfrac{e^{-.3t}}{-.3} + C = 40e^{-.3t} + C$

At initial time, $t = 0$ & Temperature $T(0) = 70$.

So, $40e^{-.3\cdot 0} + C = 70$, or

$40 + C = 70$, so $C = 30$.

$T(t) = 40e^{-.3t} + 30$.

The rest of this Examination is to be done using the "scan-tron" sheet. Please write your work on this paper.

1. $\int \frac{1}{\sqrt[4]{x^3}}dx =$

$\int \frac{1}{x^{3/4}}dx = \int x^{-3/4}\,dx = 4x^{1/4}+C$

(a). $\frac{3}{5}x^{\frac{5}{3}} + C.$
(b). $4x^{\frac{1}{4}} + C.$
(c). $-3x^{\frac{-1}{3}} + C.$
(d). $\frac{-2}{3}x^{\frac{-5}{3}} + C.$

2. The area under the curve $f(x) = x^2$ and above the x−axis, from $x = -2$ to $x = 2$, is:

$\int_{-2}^{2} x^2\,dx = \frac{x^3}{3}\Big|_{-2}^{2} = \frac{8}{3} - \left(-\frac{8}{3}\right) =$
$16/3$

(a). 0 square units.
(b). 16/3 square units.
(c). 8 square units.
(d). 16 square units.

3. $\int (x + 1)(x - 2)dx =$:

$\int (x^2 - x - 2)\,dx =$
$\frac{x^3}{3} - \frac{x^2}{2} - 2x + C$

(a). $(x^2/2 + 2x))(x^2/2 - 2x) + C.$
(b). $2x - 1 + C.$
(c). $x^3/3 - 2x + C.$
(d). $x^3/3 - x^2/2 - 2x + C.$

4. $\int_1^{e^2} \frac{2}{3t}dt =$:

$\frac{2}{3}\int_1^{e^2} \frac{1}{t}\,dt = \frac{2}{3}\ln t\Big|_1^{e^2} = \frac{2}{3}\left(\ln e^2 - \ln 1\right)$
$= \frac{2}{3}\left(2\ln e - \ln 1\right) = \frac{4}{3}$

(a). 4/3.
(b). 2/3.
(c). −1/3.
(d). 6.

5. Let $h(u) = (3u^4 - 3u + 2)^3$. Then $h'(u) =$:

$3(3u^4 - 3u + 2)^2(12u^3 - 3)$

(a). $3(3u^4 - 3u + 2)^2.$
(b). $3(12u^3 - 3)^2.$
(c). $12u^3 - 3.$
(d). $3(3u^4 - 3u + 2)^2(12u^3 - 3).$

6. You invest \$1,000 today, at 20% interest per year, with continuous compounding. In how many years will your investment triple to \$3,000?
(a). 6.03 years.
(b). 5.63 years.
(c). 6.52 years.
(d). 5.49 years.

$e^{.2x} = 3$, or $.2x = \ln 3$, so

$x = \frac{\ln 3}{.2} = 5.49$

7. What is the derivative of $f(x) = 4e^{x^2}$?
(a). $8e^{x^2}$.
(b). $8xe^{x^2}$.
(c). $8xe$.
(d). $8x^2e^{x^2-1}$.

$f'(x) = 4 \cdot e^{x^2} \cdot 2x$

$= 8xe^{x^2}$