A

Intuitive Calculus 11012 Examination 1
February 11, 2010 Richard M. Aron

NAME.....

Directions: Please answer questions A, B, C, and D in the space provided. ***Please write extremely neatly.*** **The rest of the Examination questions are to be answered on the "scan-tron" papers,** ***but you must show your work on this paper—even for the "scan-tron" part. Good luck!***

Product Rule: if $p(x) = f(x) \cdot g(x)$, then $p'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$.

Quotient Rule: if $q(x) = \frac{f(x)}{g(x)}$, then $q'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$.

A. Let $f(x) = x^2 - 5x + 5$.

(a). Calculate the slope of the tangent to f at the point $x = 2$.

$f'(2)$ = slope.

$f'(x) = 2x - 5$, so $f'(2) = -1$ = slope

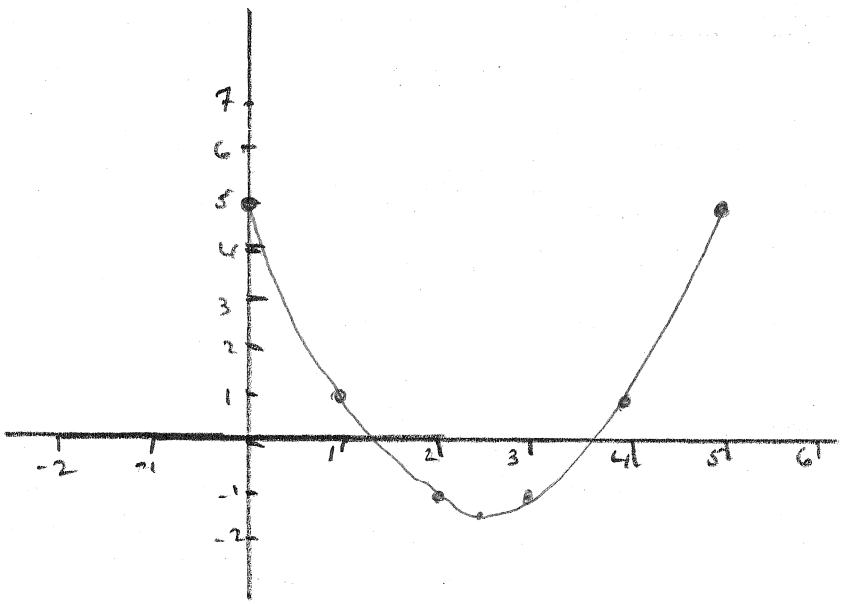
(b). Calculate the equation of the tangent line to f at $x = 2$.

If $x = 2$, $f(x) = f(2) = -1$

So, $\frac{y - (-1)}{x - 2} = -1$, that is, $y + 1 = -x + 2$, or

$y = -x + 1$

(c). In the area provided below, graph the function $f(x)$ where x goes from $x = 0$ to $x = 5$.



B. By imagining tangent lines at points P_1, P_2, and P_3 in the graph below, state whether the slopes are positive, zero, or negative at these points.

At P_1, slope is $> 0, = 0$, or < 0. Which?

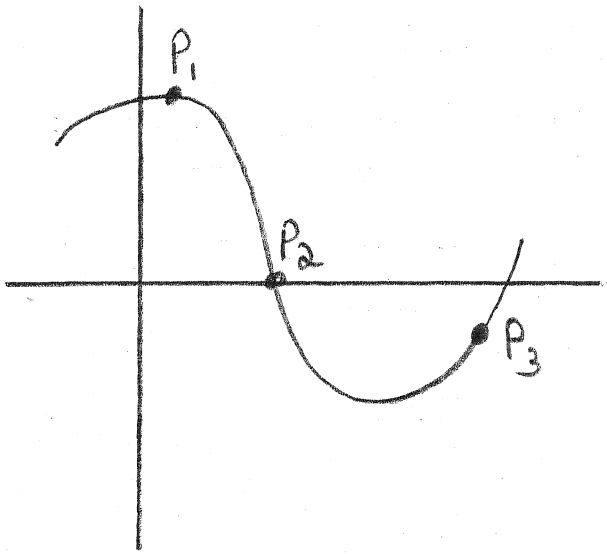
$= 0$

At P_2, slope is $> 0, = 0$, or < 0. Which?

< 0

At P_3, slope is $> 0, = 0$, or < 0. Which?

> 0



C. Let $f(x) = 3x^2 + x - 1$.

(a). Calculate the average rate of change of f between the $x-$values $x = 2$ and $x = 4$.

$\frac{f(4) - f(2)}{2} = \frac{51 - 13}{2} = 19$

(b). Calculate the average rate of change of f between the $x-$values $x = 2$ and $x = 3$.

$\frac{f(3) - f(2)}{1} = \frac{29 - 13}{1} = 16$

(c). Calculate

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}.$$

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{[3(2+h)^2 + (2+h) - 1] - [3 \cdot 2^2 + 2 - 1]}{h}$$

$$= \lim_{h \to 0} \frac{[3(4h + h^2) + h]}{h} = \lim_{h \to 0} 12 + h + 1 = 13$$

D. In each case, find the derivative of f. (*There is no need to simplify.*)

(a). $f(x) = x^{100}$. $f'(x) = 100x^{99}$

(b). $f(x) = x^{\frac{1}{3}}$. $f'(x) = \frac{1}{3}x^{-2/3}$

(c). $f(x) = \frac{x^2+5x}{\sqrt{x}}$. $f(x) = \frac{x^2+5x}{x^{1/2}}$.

$$f'(x) = \frac{x^{\frac{1}{2}}[2x+5] - [x^2+5x]\frac{1}{2}x^{-1/2}}{(x^{1/2})^2}$$

$$= \frac{x^{\frac{1}{2}}[2x+5] - \frac{1}{2}[x^2+5x]x^{-1/2}}{x}$$

(d). $f(x) = (x^2+1)(x^{10}+3x)$.

$$f'(x) = 2x(x^{10}+3x) + (x^2+1)(10x^9+3)$$

The rest of this Examination is to be done using the "scan-tron" sheet. Please write your work on this paper.

1. Find $\lim_{x\to 0^-} f(x)$ when $f(x) = \frac{|x|}{x}$.

   A. 1

   B. This one-sided limit is not defined.

   C. -1

   D. 0

2. Let $y = f(x) = 2^4$. Then $\frac{dy}{dx} =$:

   A. $4 \cdot 2^3$.

   B. 1.

   C. 0.

   D. 16.

   ($y = f(x) = 16$, constant function. So $\frac{dy}{dx} = f'(x) = 0$)

3. Find $\lim_{x\to -2} \frac{x^2-4}{x+2}$.

   $\frac{x^2-4}{x+2} = x-2$

   A. 4.

   B. 0.

   C. This limit is not defined.

   D. -4.

4. Suppose that f is a function whose derivative $f'(x) = 4x^3 + x - 5$. Then $f(x)$ can be:

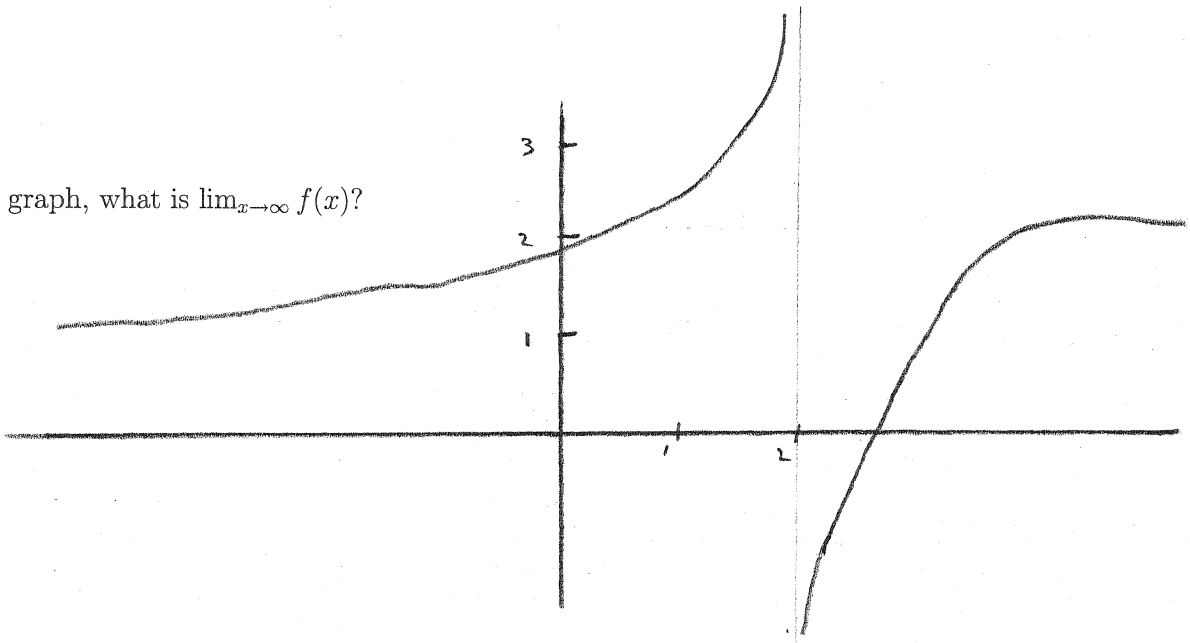
   A. $12x^2 + 1$.

   B. $x^4 + x^2$.

   C. $x^4 + \frac{1}{2}x^2 - 5x + 6$.

   D. None of the above answers.

5. From the graph, what is $\lim_{x\to\infty} f(x)$?



A. 2

B. The limit does not exist.

C. ∞

D. 1

6. Find $\lim_{h\to -1} \frac{(3+h)^3-3^3}{h}$

A. 19

B. $3 \cdot 3^2$.

C. -19

D. This limit is not defined.

7. Find $\lim_{h\to 0} \quad 3x^2 + 5xh + h^2$.

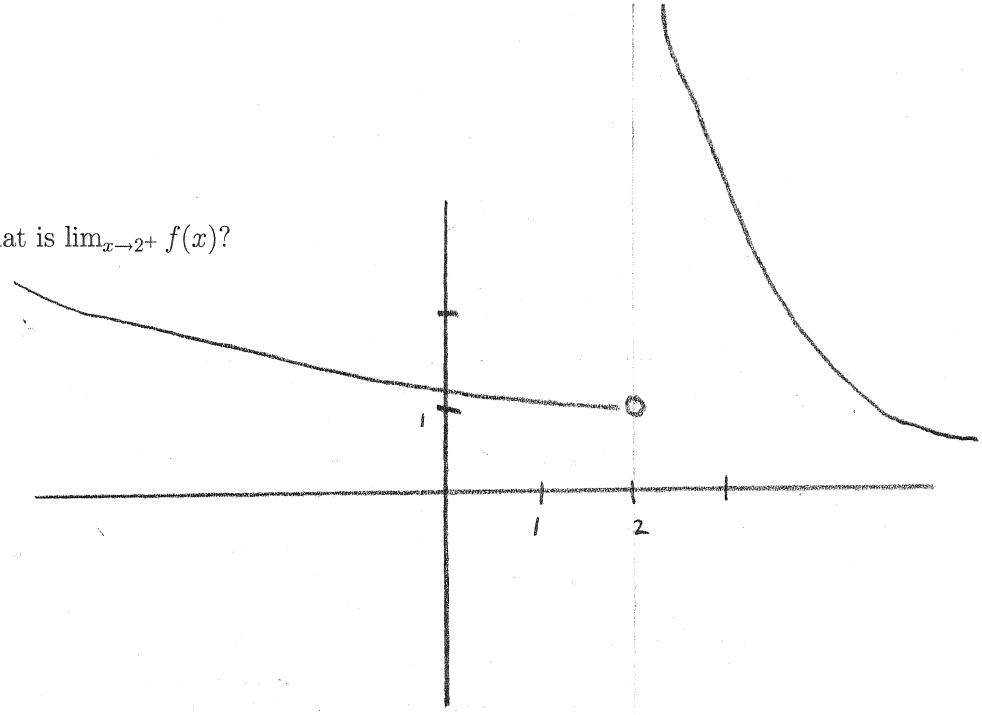
A. $3x^2 + 5x$

B. $3x^2$.

C. $6x + 5$

D. $6x$.

8. From the graph, what is $\lim_{x \to 2^+} f(x)$?



A. ∞

B. 1

C. $-\infty$

D. Since $f(2)$ is not defined, there is no answer.

9. If the cost function for a business is given by the function $C(x) = 50x^2 + 3x$, what is the margin cost function $MC(x)$?

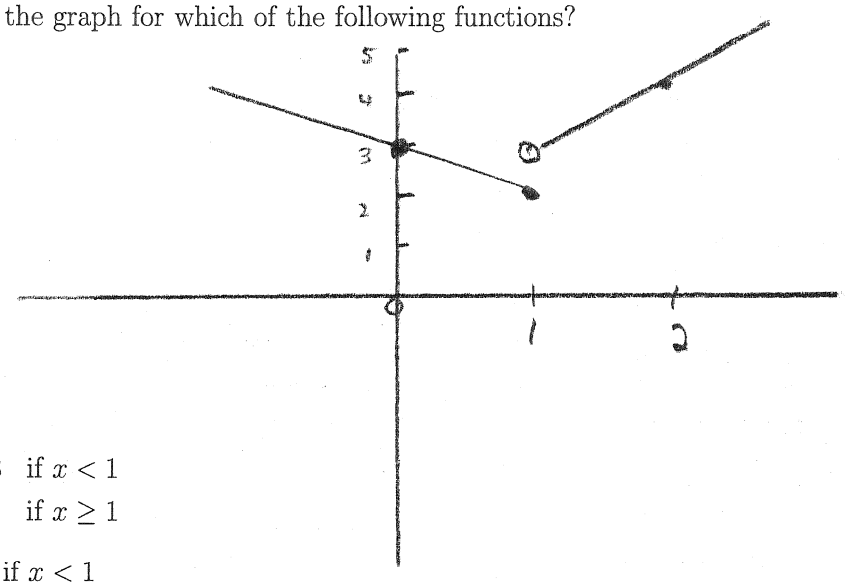
A. $MC(x) = 50x$

B. $MC(x) = 100x$

C. $MC(x) = 100x + 3$

D. $MC(x) = 103x$

10. The given graph is the graph for which of the following functions?



A. $f(x) = \begin{cases} -x+3 & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$

B. $f(x) = \begin{cases} 2 & \text{if } x < 1 \\ x+2 & \text{if } x \geq 1 \end{cases}$

C. $f(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ x+2 & \text{if } x > 1 \end{cases}$

D. $f(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ 3 & \text{if } x > 1 \end{cases}$

B

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A. Let $f(x) = x^2 - 3x + 5$.

(a). Calculate the slope of the tangent to f at the point $x = 2$.

slope $= f'(2)$.

$f'(x) = 2x - 3$

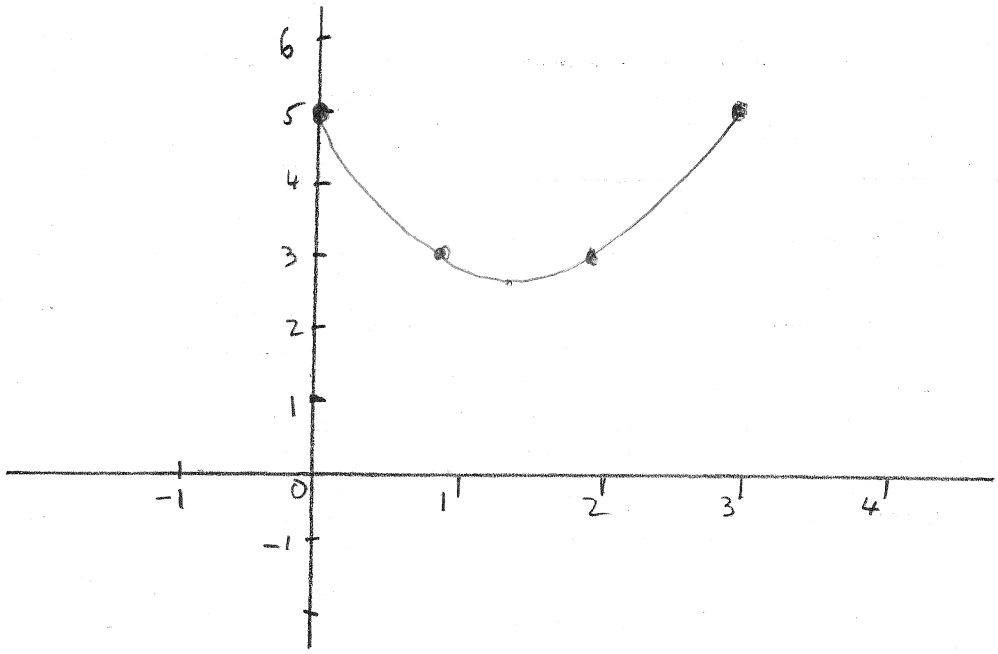
So, slope $= 1$

(b). Calculate the equation of the tangent line to f at $x = 2$.

$f(2) = 3$

$\frac{y-3}{x-2} = 1$, so $y - 3 = x - 2$, so $y = x + 1$

(c). In the area provided below, graph the function $f(x)$ where x goes from $x = 0$ to $x = 3$.



B. By imagining tangent lines at points P_1, P_2, and P_3 in the graph below, state whether the slopes are positive, zero, or negative at these points.

At P_1, slope is $> 0, = 0$, or < 0. Which?

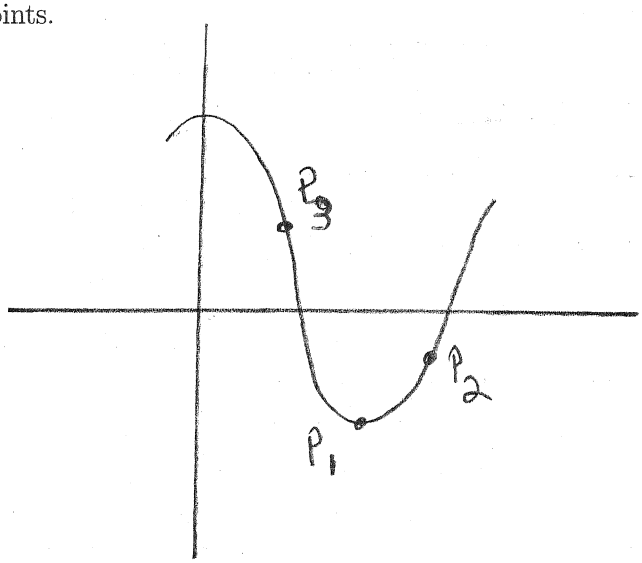
$= 0$

At P_2, slope is $> 0, = 0$, or < 0. Which?

> 0

At P_3, slope is $> 0, = 0$, or < 0. Which?

< 0



C. Let $f(x) = 3x^2 - x + 1$.

(a). Calculate the average rate of change of f between the $x-$values $x = 2$ and $x = 4$.

$\frac{f(4) - f(2)}{2} = \frac{45 - 11}{2} = 17$

(b). Calculate the average rate of change of f between the $x-$values $x = 2$ and $x = 3$.

$\frac{f(3) - f(2)}{1} = \frac{25 - 11}{1} = 14$

(c). Calculate

$$\lim_{h \to 0} \frac{f(2+h) - f(2)}{h}.$$

$$\lim_{h \to 0} \frac{[3(2+h)^2 - (2+h) + 1] - [3 \cdot 2^2 - 2 + 1]}{h}$$

$$= \lim_{h \to 0} \frac{[3(4h + h^2) - h]}{h} = \lim_{h \to 0} \frac{12h + 3h^2 - h}{h}$$

$$= \lim_{h \to 0} 12 + 3h - 1 = 11$$

D. In each case, find the derivative of f. (*There is no need to simplify.*)

(a). $f(x) = x^{50}$. $f'(x) = 50x^{49}$

(b). $f(x) = x^{\frac{1}{4}}$. $f'(x) = \frac{1}{4}x^{-3/4}$

(c). $f(x) = \frac{x^2+6x}{\sqrt{x}}$. $f(x) = \frac{x^2+6x}{x^{1/2}}$

$$f'(x) = \frac{x^{1/2}(2x+6) - (x^2+6x)\frac{1}{2}x^{-1/2}}{(x^{1/2})^2}$$

$$= \frac{x^{\frac{1}{2}}(2x+6) - (x^2+6x)\frac{1}{2}x^{-1/2}}{x}$$

(d). $f(x) = (x^3+2)(x^{10}+3x)$.

$$f'(x) = 3x^2(x^{10}+3x) + (x^3+2)(10x^9+3)$$

The rest of this Examination is to be done using the "scan-tron" sheet. Please write your work on this paper.

1. Find $\lim_{x\to 0^+} f(x)$ when $f(x) = \frac{|x|}{x}$.

(A. 1)

B. -1.

C. This one-sided limit is not defined

D. 0

2. Let $y = f(x) = 5^3$. Then $\frac{dy}{dx} =$:

A. $3 \cdot 5^2$.

B. 1.

C. 75.

D. 0.

$y = 125$, a constant.
So, $y' = 0$

3. Find $\lim_{x\to -3} \frac{x^2-9}{x+3}$.

$\frac{x^2-9}{x+3} = \frac{(x-3)(x+3)}{x+3} = x-3$

(A. -6.)

B. 6.

C. This limit is not defined.

D. 0.

4. Suppose that f is a function whose derivative $f'(x) = 5x^4 + x + 1$. Then $f(x)$ can be:

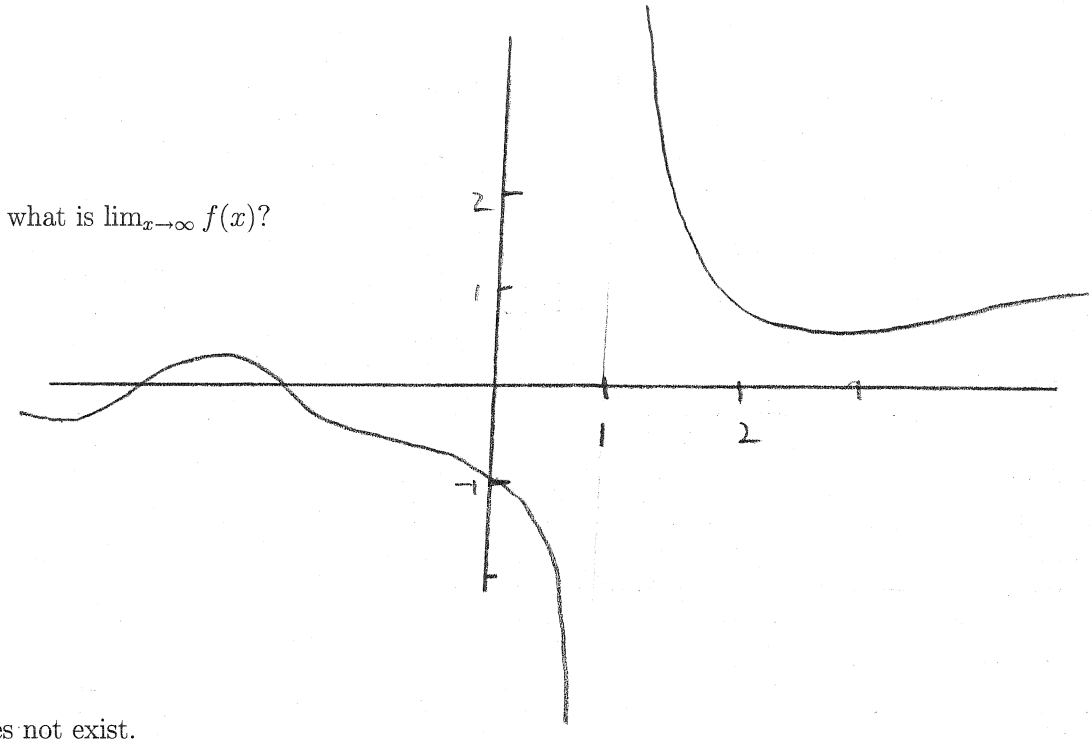
A. $20x^3 + 1$.

(B. $x^5 + \frac{1}{2}x^2 + x + 6$.)

C. $x^5 + x^2 + x^2$.

D. None of the above answers.

5. From the graph, what is $\lim_{x\to\infty} f(x)$?



A. 2

B. The limit does not exist.

C. ∞

D. 1

6. Find $\lim_{h\to -1} \frac{(2+h)^3-2^3}{h}$

A. $3 \cdot 2^2$.

B. -7.

C. This limit is not defined.

D. 7.

7. Find $\lim_{h\to 0} \ 4x^2 + 7xh + h^2$.

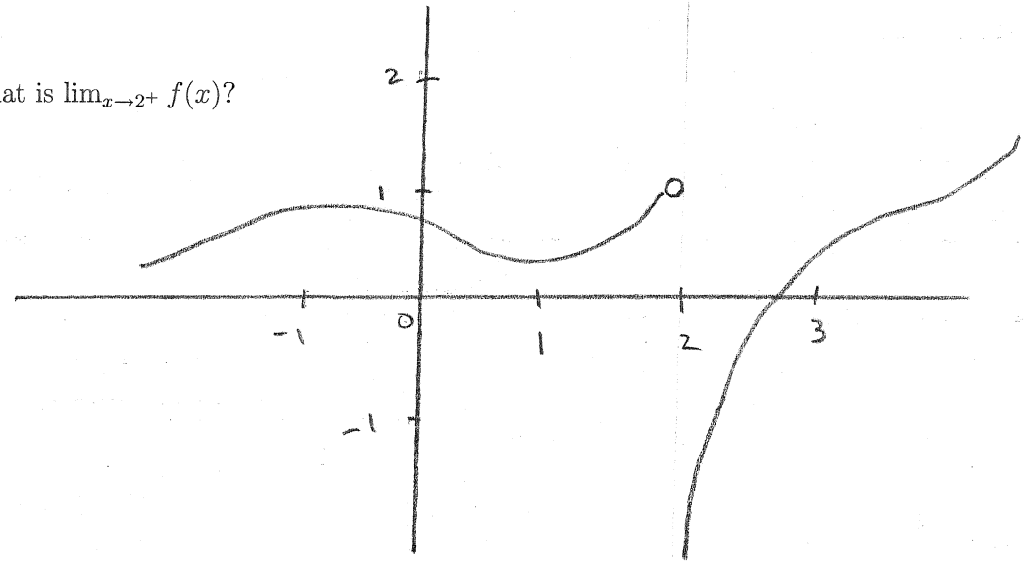
A. $4x^2 + 7x$

B. $8x$.

C. $8x + 7$

D. $4x^2$.

8. From the graph, what is $\lim_{x \to 2^+} f(x)$?



A. ∞

B. 1

C. $-\infty$

D. Since $f(2)$ is not defined, there is no answer.

9. If the cost function for a business is given by the function $C(x) = 20x^2 + 12x$, what is the margin cost function $MC(x)$?

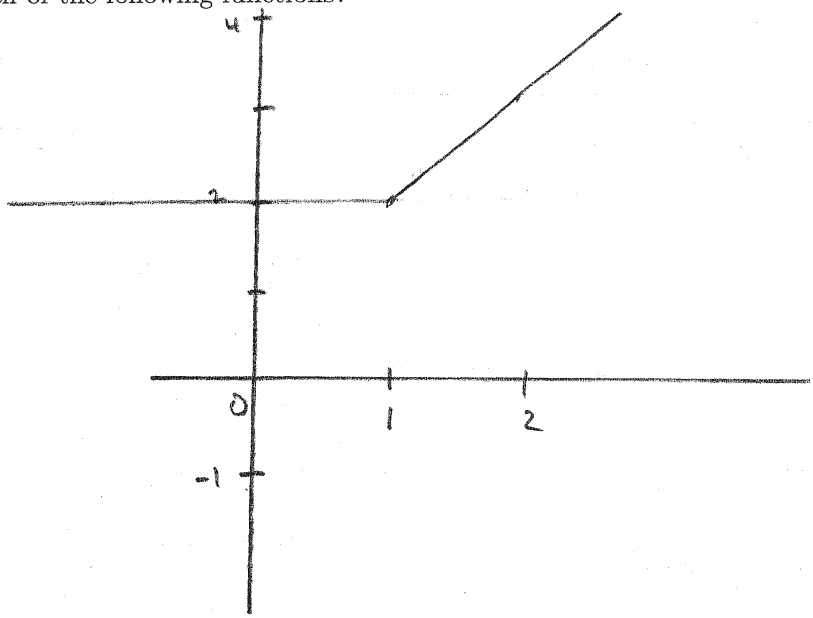
A. $MC(x) = 20x$

B. $MC(x) = 400x$

C. $MC(x) = 52x$

D. $MC(x) = 40x + 12$

10. The given graph is the graph for which of the following functions?



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C. $f(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ x+1 & \text{if } x > 1 \end{cases}$

D. $f(x) = \begin{cases} -x+3 & \text{if } x \leq 1 \\ 2 & \text{if } x > 1 \end{cases}$