A

Intuitive Calculus Mathematics 11012
Quiz 3 March 4, 2010 R. M. Aron

An airline finds that if it prices a cross-country ticket at \$800, it will sell 100 tickets per day. It estimates that each \$10 price reduction will result in 10 more tickets sold per day. Find the ticket price (and the number of tickets sold) that will maximize the airline's revenue.

**Hint: Let x = the number of price reductions.**

**1. Calculate p(x), the price per ticket with x price reductions.**

$p(x) = 800 - 10x$ (dollars)

**2. Calculate q(x), the number of tickets that are sold with x price reductions.**

$q(x) = 100 + 10x$ (tickets)

**3. The total revenue, with x price reductions, is R(x) = p(x) q(x). Use calculus to find the maximum value of R(x). Use the second derivative test to prove that your maximum really is a maximum.**

$R(x) = (800-10x)(100+10x) = 80000 - 1000x + 8000x - 100x^2$

$= 80000 + 7000x - 100x^2.$

$R'(x) = 7000 - 200x.$ Set $R'(x) = 0$, solve for x.

Get $x = 35.$ $R''(x) = -200$, so max since $R'' < 0.$

Ticket price: $800 - 10 \cdot 35 = 800 - 350 = \$450$

Number of tickets: $100 + 10 \cdot 35 = 100 + 350 = 450$ tickets.

B

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An automobile dealer expects to sell 512 cars a year. The cars cost $9000 plus a fixed charge of $1000 per delivery. If it costs $1000 to store a car for a year, find the order size and the number of orders that minimize inventory costs.

**Hint: Let x = the order size each time.**

**1.** Calculate **N(x)**, the number of orders that the dealer will place in a year.

$N(x) = \frac{512}{x}$ orders

**2.** Calculate **C(x),** the **storage cost** in a year.

$C(x) = \frac{x}{2} \cdot 1000 = 500x$ dollars

**3.** Calculate **R(x),** the **reorder cost each time..**

$R(x) = 9000x + 1000$ dollars

**4.** The **total inventory cost is T(x) = C(x) + N(x) R(x). Use calculus to find the minimum value of T(x). Use the second derivative test to prove that your minimum really is a minimum.**

$T(x) = 500x + \frac{512}{x}\left[9000x + 1000\right] = 500x + 4{,}608{,}000 + \frac{512{,}000}{x}$

$T'(x) = 500 - 512{,}000x^{-2}$. Solve for x, getting x = 32.

$T''(x) = \frac{2 \cdot 512{,}000}{x^3}$, so $T''(32) > 0$. Min.

__Order size__ that minimizes total cost:

Order size = 32 ~~orders~~ size

__Number of orders__ that minimize total cost:

Number of orders $= \frac{512}{32} =$ 16 orders