An airline finds that if it prices a cross-country ticket at $800, it will sell 100 tickets per day. It estimates that each $10 price reduction will result in 10 more tickets sold per day. Find the ticket price (and the number of tickets sold) that will maximize the airline’s revenue.

**Hint:** Let \( x \) = the number of price reductions.

1. **Calculate** \( p(x) \), the price per ticket with \( x \) price reductions.
   \[
   p(x) = 800 - 10x \quad \text{(dollars)}
   \]

2. **Calculate** \( q(x) \), the number of tickets that are sold with \( x \) price reductions.
   \[
   q(x) = 100 + 10x \quad \text{(tickets)}
   \]

3. The total revenue, with \( x \) price reductions, is \( R(x) = p(x) \cdot q(x) \). Use calculus to find the maximum value of \( R(x) \). Use the second derivative test to prove that your maximum really is a maximum.

   \[
   R(x) = (800-10x)(100+10x) = 80000 - 10000x + 8000x - 100x^2
   \]
   \[
   = 80000 + 7000x - 100x^2.
   \]
   \[
   R'(x) = 7000 - 200x.
   \]
   Set \( R'(x) = 0 \), solve for \( x \).
   Get \( x = 3.5 \).
   \[
   R''(x) = -200, \quad \text{so max since } R'' < 0.
   \]

   *Ticket price: \( 800 - 10 \cdot 3.5 = 800 - 35 = 765 \)
   
   *Number of tickets: \( 100 + 10 \cdot 3.5 = 100 + 35 = 135 \) tickets.
An automobile dealer expects to sell 512 cars a year. The cars cost $9000 plus a fixed charge of $1000 per delivery. If it costs $1000 to store a car for a year, find the order size and the number of orders that minimize inventory costs.

**Hint:** Let $x =$ the order size each time.

1. Calculate $N(x)$, the number of orders that the dealer will place in a year.
   \[ N(x) = \frac{512}{x}\text{ orders} \]

2. Calculate $C(x)$, the storage cost in a year.
   \[ C(x) = \frac{x}{2} \cdot 1000 = 500x\text{ dollars} \]

3. Calculate $R(x)$, the reorder cost each time.
   \[ R(x) = 9000x + 1000\text{ dollars} \]

4. The total inventory cost is $T(x) = C(x) + N(x)R(x)$. Use calculus to find the minimum value of $T(x)$. Use the second derivative test to prove that your minimum really is a minimum.
   \[ T(x) = 500x + \frac{512}{x} \left[ 9000x + 1000 \right] = 500x + 4600000 + \frac{512000}{x} \]
   \[ T'(x) = 500 - \frac{512000}{x^2}, \text{ solve for } x, \text{ getting } x = 32 \]
   \[ T''(x) = \frac{2 \cdot 512000}{x^3}, \text{ } 0, \text{ } T''(32) > 0, \text{ } \text{Min} \]

**Order size** that minimizes total cost:

Order size $= 32 \text{ order size}$

**Number of orders** that minimize total cost:

Number of orders $= \frac{512}{32} = 16 \text{ orders}$