

**II. Limits.**

1. Find the following limits.

$$\text{a) } \lim_{x \rightarrow 4} \sqrt{x^2 + x + 5} \quad \text{b) } \lim_{x \rightarrow -1} \frac{3x^3 - 3x}{2x^2 + 2x} \quad \text{c) } \lim_{h \rightarrow 0} \frac{6xh^2 - x^2h}{h}$$

$$2. \quad f(x) = \begin{cases} 2x - 7 & \text{if } x \leq 5 \\ 3 - x & \text{if } x < 5 \end{cases}$$

Find  $\lim_{x \rightarrow 5^-} f(x)$ ,  $\lim_{x \rightarrow 5^+} f(x)$ ,  $\lim_{x \rightarrow 5} f(x)$ .**III. Derivatives**

1. Find the derivative of each function.

$$\text{a) } f(x) = 4x^5 + \frac{x^4}{4} - 2\sqrt{x} + \frac{8}{\sqrt{x}} - \sqrt[3]{x^2} + x - 1, \quad \text{b) } f(x) = \frac{x^2 + 1}{x^2 - 1}, \quad \text{c) } f(x) = (3x^2 - 5x + 1)^4,$$

$$\text{d) } f(x) = \sqrt{x^2 - 5x - 10}, \quad \text{e) } f(x) = \left(\frac{x+4}{x}\right)^5, \quad \text{f) } f(x) = 2x^3 - 3xe^{2x}, \quad \text{g) } f(x) = x \ln x - x,$$

$$\text{h) } f(x) = \ln \sqrt{x^2 + 1}, \quad \text{i) } f(x) = (3x + 1)e^{x^3}, \quad \text{j) } f(x) = [2x^3 - (x^2 - 5)^4]^3, \quad \text{k) } f(x) = \ln(xe^{5x}).$$

**IV. Graphing**

1. Graph each function "by hand", showing all relative extreme points and inflection points.

$$\text{a) } f(x) = x^3 + 3x^2 - 9x - 7, \quad \text{b) } f(x) = x^4 + 4x^3 + 17.$$

2. Find the absolute extreme values of each function on the given interval.

$$\text{a) } f(x) = 2x^3 - 24x \text{ on } [0, 5], \quad \text{b) } f(x) = x^4 - 4x^3 + 4x^2 + 1 \text{ on } [0, 10].$$

**IV. Applications of Exponential Functions, Logarithmic Functions and Derivatives.**

1. Find the value of \$5000 invested for 7 years at 4.75% interest compounded

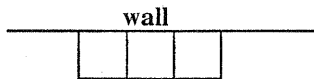
a) monthly, b) continuously, c) find how soon will the deposit triple in both cases, d) find how soon will the amount of money be \$8000?

3. A rocket can rise to a height of  $h(t) = t^3 + 0.5t^2$  feet in  $t$  seconds. Find its velocity and acceleration 10 seconds after it is launched.4. Find the slope of a tangent line to the graph of function  $f(x) = (x^2 - 1)e^{2x}$  at  $x = 0$ .5. Suppose that for a group of 10,000 people, the number who survive to age  $x$  is  $N(x) = 500\sqrt{100 - x}$ . Find how many people survive to 96 years old and instantaneous rate of change of survivors at this age.

6. A company's cost function is  $C(x) = 5x + 100$  dollars, where  $x$  is the number of units. a) Find the average cost function; b) Find the marginal average cost function; c) Evaluate  $AMC(x)$  at  $x = 20$  and interpret your answer.

### V. Optimization.

1. A homeowner wants to enclose three adjacent rectangular pens of equal size along a straight wall. If the side along the wall needs no fence, what is the largest area that can be enclosed using only 240 feet of fence?



2. Country Motorbikes Incorporated finds that it costs \$200 to produce each motorbike, and that fixed costs are \$1500 per day. The price function is  $p(x) = 600 - 5x$ , where  $p$  is the price (in dollars) at which exactly  $x$  motorbikes will be sold. Find the quantity Country Motorbikes should produce and the price it should charge to maximize profit. Also find the maximum profit.
3. An open-top box with a square base is to have a volume of exactly 500 cubic inches. Find the dimensions of the box that can be made with the smallest amount of materials.
4. The given function is a company's price function, where  $x$  is the quantity (in thousands) that will be sold at price  $p$  dollars. Find the revenue function. Find also the quantity and price that will maximize revenue.

a)  $p = 200e^{-0.25x}$ ;

b)  $p = 5 - \ln x$

### VI. Integrals.

2. Find each integral.

a)  $\int (12x^3 + 6x - 3)dx$ , b)  $\int (10\sqrt[3]{x^2} - \frac{4}{\sqrt{x}})dx$ , c)  $\int (9x^2 + \frac{1}{x} + e^{-x})dx$ ,

d)  $\int \frac{(x-4)^2}{x}dx$ , e)  $\int (x^5 - 4) \cdot 5x^4 dx$ , f)  $\int x^3 \sqrt{x^4 - 1} dx$ , g)  $\int \frac{dx}{1-2x}$ ,

h)  $\int \frac{t-2}{(t^2-4t+1)^2} dt$ , i)  $\int xe^{2x^2} dx$ , j)  $\int \frac{3e^{2x}}{e^{2x}-1} dx$ , k)  $\int \frac{(\ln x)^3}{2x} dx$ .

3. Evaluate each definite integral.

a)  $\int_1^4 (3x^2 - 4x + 5)dx$ ,    b)  $\int_1^5 \frac{dx}{x}$ ,    c)  $\int_0^4 e^{\frac{x}{2}} dx$ ,    d)  $\int_0^4 \frac{w}{\sqrt{25 - w^2}} dw$ .

## VII. Application of Integrals.

2. Find the area under the curve between the given values of  $x$ .

a)  $f(x) = 6x^2 - 1$ , from  $x = 1$  to  $x = 2$ ;

b)  $f(x) = e^{x/2}$ , from  $x = 0$  to  $x = 4$ ;

c)  $f(x) = \frac{2}{x}$ , from  $x = 1$  to  $x = 100$ .

3. Find the area bounded by each pair of curves.

a)  $f(x) = x^2$  and  $g(x) = x$ ;    b)  $f(x) = 12x - 3x^2$  and  $g(x) = 6x - 24$ .

4. The price of a share of stock is expected to be  $28e^{0.01x}$  dollars, where  $x$  is the number of weeks from now. Find the average price over the next year (week 0 to week 52).

4. A culture of bacteria is growing at the rate of  $20e^{0.4t}$  cells per day, where  $t$  is a number of days since the culture was started. If the culture began with 40 cells, find the formula for the total number of cells in the culture after  $t$  days. Also find how long will it take to reach 1000 cells in the culture.

6. An epidemic is spreading at the rate of  $12e^{0.2t}$  new cases per day, where  $t$  is the number of days since the epidemic began. Find the total number of new cases in the first 20 days of the epidemic.