Exponential and Logarithmic Functions

Do not use a calculator.

1. Let \( f(x) = 2^x \).

   (a) Give the domain and range in interval notation.
      i. \( \text{dom} \ (2^x) = \)
      ii. \( \text{range} \ (2^x) = \)

   (b) Give the ordered pairs for all intercepts of \( f \).
      i. \( y \)-intercepts:
      ii. \( x \)-intercepts:

   (c) Complete the table of values.
      \[
      \begin{array}{c|c}
        x & f(x) \\
        \hline
        -3 & \\
        -2 & \\
        -1 & \\
        0 & \\
        1 & \\
        2 & \\
        3 & \\
      \end{array}
      \]

   (d) Evaluate each limit.
      i. \( \lim_{{x \to -\infty}} 2^x = \)
      ii. \( \lim_{{x \to \infty}} 2^x = \)

   (e) Give the equations of all asymptotes of \( f \).
      i. vertical asymptotes:
      ii. horizontal asymptotes:

   (f) Sketch the graph \( y = f(x) \) on the coordinate system on the graph page. Label all points computed above with their ordered pairs and all asymptotes with their equations.

   (g) Give the formula for the inverse of \( f \).
      \( f^{-1}(x) = \)

   (h) Give the domain and range in interval notation.
      i. \( \text{dom} \ (\log_2 x) = \)
      ii. \( \text{range} \ (\log_2 x) = \)

   (i) Give the ordered pairs for all intercepts of the graph \( y = \log_2 x \).
      i. \( y \)-intercepts:
      ii. \( x \)-intercepts:

   (j) Evaluate each limit.
      i. \( \lim_{{x \to 0^+}} \log_2 x = \)
      ii. \( \lim_{{x \to \infty}} \log_2 x = \)

   (k) Give the equations of all asymptotes of the graph \( y = \log_2 x \).
      i. vertical asymptotes:
      ii. horizontal asymptotes:

   (l) Sketch the graph \( y = \log_2 x \) on the same coordinate system as \( y = 2^x \). Label at least 5 points with their ordered pairs and all asymptotes with their equations.
2. Let \( f(x) = \left(\frac{1}{2}\right)^x \).

(a) Give the domain and range in interval notation.
   i. \( \text{dom} \left( \left(\frac{1}{2}\right)^x \right) = \)
   ii. \( \text{range} \left( \left(\frac{1}{2}\right)^x \right) = \)

(b) Give the ordered pairs for all intercepts of \( f \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(c) Complete the table of values.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
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<td>-2</td>
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<td>2</td>
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<td>3</td>
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</tbody>
</table>

(d) Evaluate each limit.
   i. \( \lim_{x \to -\infty} \left(\frac{1}{2}\right)^x = \)
   ii. \( \lim_{x \to \infty} \left(\frac{1}{2}\right)^x = \)

(e) Give the equations of all asymptotes of \( f \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(f) Sketch the graph \( y = f(x) \) on the coordinate system on the graph page. Label all points computed above with their ordered pairs and all asymptotes with their equations.

(g) Give the formula for the inverse of \( f \).
   \( f^{-1}(x) = \)

(h) Give the domain and range in interval notation.
   i. \( \text{dom} \left( \log_{1/2} x \right) = \)
   ii. \( \text{range} \left( \log_{1/2} x \right) = \)

(i) Give the ordered pairs for all intercepts of the graph \( y = \log_{1/2} x \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(j) Evaluate each limit.
   i. \( \lim_{x \to 0^+} \log_{1/2} x = \)
   ii. \( \lim_{x \to \infty} \log_{1/2} x = \)

(k) Give the equations of all asymptotes of the graph \( y = \log_{1/2} x \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(l) Sketch the graph \( y = \log_{1/2} x \) on the same coordinate system as \( y = \left(\frac{1}{2}\right)^x \). Label at least 5 points with their ordered pairs and all asymptotes with their equations.
3. Let \( f(x) = 4^x \).

(a) Give the domain and range in interval notation.
   i. \( \text{dom} (4^x) = \)
   ii. \( \text{range} (4^x) = \)

(b) Give the ordered pairs for all intercepts of \( f \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(c) Complete the table of values.

<table>
<thead>
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<tbody>
<tr>
<td>-2</td>
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<tr>
<td>-1</td>
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<td>-1/2</td>
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<tr>
<td>1</td>
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<td>2</td>
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</tbody>
</table>

(d) Evaluate each limit.
   i. \( \lim_{x \to -\infty} 4^x = \)
   ii. \( \lim_{x \to \infty} 4^x = \)

(e) Give the equations of all asymptotes of \( f \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(f) Sketch the graph \( y = f(x) \) on the coordinate system on the graph page. *Label all points computed above with their ordered pairs and all asymptotes with their equations.*

(g) Give the formula for the inverse of \( f \).
   \( f^{-1}(x) = \)

(h) Give the domain and range in interval notation.
   i. \( \text{dom} (\log_4 x) = \)
   ii. \( \text{range} (\log_4 x) = \)

(i) Give the ordered pairs for all intercepts of the graph \( y = \log_4 x \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(j) Evaluate each limit.
   i. \( \lim_{x \to 0^+} \log_4 x = \)
   ii. \( \lim_{x \to \infty} \log_4 x = \)

(k) Give the equations of all asymptotes of the graph \( y = \log_4 x \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(l) Sketch the graph \( y = \log_4 x \) on the same coordinate system as \( y = 4^x \). *Label at least 5 points with their ordered pairs and all asymptotes with their equations.*
4. Let \( f(x) = \left(\frac{1}{4}\right)^x \).

(a) Give the domain and range in interval notation.
   i. \( \text{dom} \left(\frac{1}{4}\right)^x = \)
   ii. \( \text{range} \left(\frac{1}{4}\right)^x = \)

(b) Give the ordered pairs for all intercepts of \( f \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(c) Complete the table of values.

<table>
<thead>
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<tr>
<td>(1)</td>
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<tr>
<td>(2)</td>
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</tbody>
</table>

(d) Evaluate each limit.
   i. \( \lim_{x \to -\infty} \left(\frac{1}{4}\right)^x = \)
   ii. \( \lim_{x \to \infty} \left(\frac{1}{4}\right)^x = \)

(e) Give the equations of all asymptotes of \( f \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(f) Sketch the graph \( y = f(x) \) on the coordinate system on the graph page. Label all points computed above with their ordered pairs and all asymptotes with their equations.

(g) Give the formula for the inverse of \( f \).
\( f^{-1}(x) = \)

(h) Give the domain and range in interval notation.
   i. \( \text{dom} \left(\log_{1/4} x\right) = \)
   ii. \( \text{range} \left(\log_{1/4} x\right) = \)

(i) Give the ordered pairs for all intercepts of the graph \( y = \log_{1/4} x \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(j) Evaluate each limit.
   i. \( \lim_{x \to 0^+} \log_{1/4} x = \)
   ii. \( \lim_{x \to \infty} \log_{1/4} x = \)

(k) Give the equations of all asymptotes of the graph \( y = \log_{1/4} x \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(l) Sketch the graph \( y = \log_{1/4} x \) on the same coordinate system as \( y = \left(\frac{1}{4}\right)^x \). Label at least 5 points with their ordered pairs and all asymptotes with their equations.
5. Let \( f(x) = e^x \).

(a) Give the domain and range in interval notation.
   i. \( \text{dom}(e^x) = \)
   ii. \( \text{range}(e^x) = \)

(b) Give the ordered pairs for all intercepts of \( f \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(c) Evaluate each limit.
   i. \( \lim_{x \to -\infty} e^x = \)
   ii. \( \lim_{x \to \infty} e^x = \)

(d) Give the equations of all asymptotes of \( f \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(e) Sketch the graph \( y = f(x) \) on the coordinate system on the graph page. **Label at least 5 points with their ordered pairs and all asymptotes with their equations.** You may use the following estimates:

\[
e \approx 2.7 \\
e^2 \approx 7.4
\]

(f) Give the formula for the inverse of \( f \).
\( f^{-1}(x) = \)

(g) Give the domain and range in interval notation.
   i. \( \text{dom}(\ln x) = \)
   ii. \( \text{range}(\ln x) = \)

(h) Give the ordered pairs for all intercepts of the graph \( y = \ln x \).
   i. \( y \)-intercepts:
   ii. \( x \)-intercepts:

(i) Evaluate each limit.
   i. \( \lim_{x \to 0^+} \ln x = \)
   ii. \( \lim_{x \to \infty} \ln x = \)

(j) Give the equations of all asymptotes of the graph \( y = \ln x \).
   i. vertical asymptotes:
   ii. horizontal asymptotes:

(k) Sketch the graph \( y = \ln x \) on the same coordinate system as \( y = e^x \). **Label at least 5 points with their ordered pairs and all asymptotes with their equations.**
1. \( y = 2^x \) and \( y = \log_2 x \).

2. \( y = \left(\frac{1}{2}\right)^x \) and \( y = \log_{1/2} x \).
3. $y = 4^x$ and $y = \log_4 x$.

4. $y = \left(\frac{1}{4}\right)^x$ and $y = \log_{1/4} x$. 
5. $y = e^x$ and $y = \ln x$. 