EXAM 2—Friday, June 20, 2003

Do the easier questions first. Show all your work. GOOD LUCK!

1. (15 points) Find the derivatives of the following functions. DO NOT SIMPLIFY YOUR ANSWERS! However, it might help to simplify the original function before differentiating.

   (a) \( f(x) = \sqrt{\frac{2x}{x+1}} \)

   \[ f'(x) = \]

   (b) \( f(x) = \frac{1}{(5x^4 - 6x^2 + 13)^{20}} \)

   \[ f'(x) = \]

   (c) \( f(x) = (2x + 1)^{100} (3x - 5) \)

   \[ f'(x) = \]

2. (10 points) Find each derivative requested. Use the notation indicated in each problem.

   (a) Find \( f^{(4)}(x) \) where \( f(x) = x^5 - x^3 + x^2 + 1 \)

   (b) Find \( \frac{d^2}{dx^2} \sqrt{x^3} \bigg|_{x=1/16} \)
3. (10 points) A rocket can rise to a height of \( h(t) = t^3 + 0.5t^2 \) feet in \( t \) seconds. Include appropriate units with your answers.

   (a) Find the velocity \( v(t) \) of the rocket after \( t \) seconds.

   (b) Find the velocity of the rocket after 10 seconds

   (c) Find the acceleration \( a(t) \) of the rocket after \( t \) seconds

   (d) Find the acceleration of the rocket after 10 seconds

4. (10 points) A company's cost function is \( C(x) = 20 + 3x + \frac{54}{\sqrt{x}} \) dollars.

   (a) Find the marginal cost function.

   (b) Find the marginal cost at \( x = 9 \) and interpret your answer in plain English.
5. (20 points) Answer the following about the function $f$ which is depicted below.

(a) Give the ordered pairs for all of the following points on the graph of $f$. Write “none” if there aren’t any. (Give approximations to the nearest 0.5 based on the graph.)

i. $x$-intercepts: ______________  
   v. relative maximum points: ______________

ii. $y$-intercepts: ______________  
   vi. relative minimum points: ______________

iii. critical points: ______________  
   vii. absolute maximum points: ______________

iv. inflection points: ______________  
   viii. absolute minimum points: ______________

(b) Give the intervals where the function $f$ has the property listed. (Use interval notation. Give approximations to the nearest 0.5, consistent with your answers above.)

i. $f$ is positive: ______________

ii. $f$ is negative: ______________

iii. $f$ is increasing: ______________

iv. $f$ is decreasing: ______________

v. $f$ is concave up: ______________

vi. $f$ is concave down: ______________
6. (15 points) Suppose the function \( h \) has the following properties.

\[
\begin{align*}
  h(-1) &= -2 & h'(2) &= 0 & h''(2) &= 0 \\
  h'(-1) & \text{is undefined} & h'(2) &= 0 \\
\end{align*}
\]

(a) Fill in the charts with the phrases “increasing”, “decreasing”, “concave up”, or “concave down”.

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<thead>
<tr>
<th>interval</th>
<th>((-\infty, -1))</th>
<th>((-1, 2))</th>
<th>((2, \infty))</th>
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<tbody>
<tr>
<td>( h'(x) )</td>
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(b) Give the \( x \)-coordinates for all of the following. (Write “none” if there aren’t any.)

Critical points of \( f \): \( x = \underline{\phantom{000}} \)
Relative maximum points of \( f \): \( x = \underline{\phantom{000}} \)
Inflection points of \( f \): \( x = \underline{\phantom{000}} \)
Relative minimum points of \( f \): \( x = \underline{\phantom{000}} \)

(c) Sketch a possible graph of the function \( h \). Indicate all critical points and inflection points as well as other important behavior very clearly.
7. (20 points) Consider the function \( f(x) = x^3 - 3x + 2 \). To get credit for the graph, you must convince me that you did this by hand. Show your reasoning.

(a) Find the \( y \)-intercept of the graph of \( f \). (Express it as an ordered pair.)

(b) Find and factor \( f'(x) \).

(c) Find the critical numbers of \( f \).

(d) Make a sign chart for \( f' \) and indicate on which intervals the original function \( f \) is increasing and on which intervals \( f \) is decreasing.

(e) Find both coordinates of all relative maximum points and relative minimum points of \( f \), indicating which are which. (Express them as ordered pairs.)

(f) Find \( f''(x) \).

(g) Make a sign chart for \( f'' \) and indicate on which intervals the original function \( f \) is concave up and on which intervals \( f \) is concave down.

(h) Find both coordinates of all inflection points of \( f \). (Express them as ordered pairs.)

(i) Carefully sketch the graph \( y = f(x) \). Plot and label all points found above. Indicate all important aspects of the graph clearly.
Bonus. (10 points) Find the derivative, \( f'(x) \), for the function \( f(x) = \sqrt{3 + \sqrt{2 + \sqrt{1 + 4x}}} \).