KENT STATE UNIVERSITY

Fall 2015 Course Announcement

ST: The Mathematics of Social Choice

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Math 49995-002 CRN: 15605 / Math 57091-002 CRN: 15622

Days: MWF Time: 2:15pm–3:05pm Room: 111 Henderson Hall

Description: Social choice theory is a framework for aggregating the preferences of individuals to produce a collective decision or distribution of goods. This course will cover the mathematical analysis of democratic voting and fair division methods, including fairness criteria, paradoxes, and impossibility theorems. Students will learn to make and test conjectures and to construct rigorous deductive arguments and counter examples.

Consider the following scenario: The twenty-nine students in the Math Club are electing a president. The four candidates are Ann, Bob, Cate, and Don. Here is a summary of the ballots for the election.

# voters	**11**	**7**	**7**	**3**	**1**
1st place	Ann	Cate	Don	Bob	Cate
2nd place	Bob	Bob	Cate	Don	Don
3rd place	Cate	Don	Bob	Cate	Bob
4th place	Don	Ann	Ann	Ann	Ann

This means that 11 Math Club members submitted ballots ranking Ann in first place, Bob in second place, Cate in third place, and Don in last place; 7 members ranked the candidates Cate > Bob > Don > Ann; and so on. The students decide to declare the person with the most first-place votes to be the winner of the election. **Thus Ann becomes president** with 11 first-place votes.

However, Bob points out that a majority of the voters put Ann in *last* place, so it's not clear that this outcome really reflects the desires of the voters. He suggests a point system, where each first place vote earns a candidate 3 points; each second-place vote is worth 2 points; each third-place vote is worth 1 point; and a fourth place vote is worth 0 points. Under this system, Ann has $3 \times 11 = 33$ points; Bob has $3 \times 3 + 18 \times 2 + 8 \times 1 = 53$ points; Cate has $8 \times 3 + 7 \times 2 + 14 \times 1 = 52$ points; Don has $7 \times 3 + 4 \times 2 + 7 \times 1 = 36$ points. **This makes Bob the winner**.

Cate says, "Not so fast." She observes that she could beat each of the other three candidates in a head-to-head competition. Indeed, 18 voters prefer Cate to Ann, while only 11 prefer Ann to Cate. So Cate would beat Ann 18-11. She would beat Bob 15-14, and she would beat Don 19-10. Since this makes her a Condorcet Candidate, **Cate should win the election**.

Finally, Don pipes up. He suggests a different method where the candidate with the fewest first-place votes is eliminated and the votes re-tallied, until a candidate reaches a majority (more than 50%) of the first-place votes. Using this method, Bob is eliminated first, giving Ann 11 first-place votes, Cate 8, and Don 10. Next Cate is eliminated, and **Don wins the election** with 18 first-place votes.

Who should be president of Math Club? Which choice really reflects the will of the voters? These are the sorts of questions we will consider in this course.

Intended Audience: Mathematics, Applied Mathematics, and Integrated Mathematics Majors. May also be suitable for Economics and Political Science Majors and others.

Prerequisites: A course in Linear Algebra is recommended. An ability to follow rigorous arguments, a desire to learn, and a willingness to work hard are required.

Questions? Contact Dr. Kracht at (330) 672-9093 or dkracht@kent.edu