Homework 1: Due Monday, September 29, 2014

1. Suppose T_x is a future lifetime random variable with lifetime distribution given by

$$F_x(t) = \begin{cases} \frac{t}{100-x}, & \text{for } t \in [0, 100-x) \\ 1, & \text{for } t \geq 100-x \end{cases}$$

Find and simplify an expression for each of the following.

(a) $\mathring{e}_x$ (b) $V[T_x]$ (c) $\text{median}[T_x]$

2. Suppose T_x is a future lifetime random variable whose probability density function is given by $f_x(t) = ce^{-ct}$ for all $t \geq 0$, where c is a positive constant. Find and simplify an expression for each of the following.

(a) $\mathring{e}_x$ (b) $V[T_x]$ (c) $\text{median}[T_x]$

3. Determine which of the following functions could serve as a force of mortality function μ_x. *Hint: Show whether the corresponding "survival function" could really be a survival function for a future lifetime random variable.*

(a) ax^b, where $a, b > 0$ (b) $\frac{1}{(1+x)^3}$

4. Show that, for $n \in \mathbb{Z}^+$, $e_{x:\overline{n}|} = \sum_{k=1}^{n} {}_kP_x$.

Use the attached life table extract for the remaining exercises. It will be helpful to fill in the missing columns of the table before you begin. Give answers with eight decimal digits or many as as your calculator displays.

5. Using the given life table extract, compute each of the following.

(a) d_{28} (b) p_{28} (c) q_{28}

(d) ${}_4p_{23}$ (e) ${}_5q_{20}$ (f) ${}_{2|3}q_{20}$

6. Using the given life table extract and the Uniform Distribution of Deaths assumption, compute each of the following.

(a) ${}_{0.3}p_{27}$ (b) ${}_{0.5}q_{23.2}$ (c) ${}_{0.8}q_{25.4}$ (d) ${}_{3.2}p_{21.1}$

7. Using the given life table extract and the Constant Force of Mortality assumption, compute each of the following.

(a) ${}_{0.3}p_{27}$ (b) ${}_{0.5}q_{23.2}$ (c) ${}_{0.8}q_{25.4}$ (d) ${}_{3.2}p_{21.1}$

Life Table Extract for Homework 1

Hint: A spreadsheet will make short work of filling in the rest of the table!

Age, x	l_x	d_x	p_x	q_x
20	97,741			
21	97,623			
22	97,499			
23	97,370			
24	97,240			
25	97,110			
26	96,982			
27	96,856			
28	96,730			
29	96,604			