

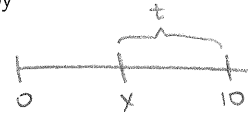
Name \_\_\_\_\_ Quiz Score: \_\_\_\_\_ /20

**Quiz 3: Monday, September 15, 2014**

Write the symbol, the formula, and your work. No credit for answers without sufficient justification. Use standard mathematical notation correctly.

Suppose the lifetime distribution function for a newborn chipmunk in my backyard is given by

$$S_0(t) = \left(1 - \frac{t}{10}\right)^{1/2} \quad F_0(t) = 1 - \left(1 - \frac{t}{10}\right)^{1/2}, \quad \text{for } 0 \leq t \leq 10.$$



1. (10 pts) Find and simplify an expression for  ${}_t p_x$ , the probability that a chipmunk aged  $x$  survives to at least age  $x + t$ .

$$\begin{aligned} {}_t p_x &= S_x(t) \\ &= \frac{S_0(x+t)}{S_0(x)} \\ &= \frac{\left(1 - \frac{x+t}{10}\right)^{1/2}}{\left(1 - \frac{x}{10}\right)^{1/2}} \\ &= \left(\frac{10-x-t}{10-x}\right)^{1/2} \\ &= \left(1 - \frac{t}{10-x}\right)^{1/2} \end{aligned}$$

2. (10 pts) Find and simplify a formula for  $\bar{e}_x$ , the complete expectation of life for a chipmunk aged  $x$ .

$$\begin{aligned} \bar{e}_x &= \int_0^{\infty} {}_t p_x dt \\ &= \int_0^{10-x} \left(1 - \frac{t}{10-x}\right)^{1/2} dt \\ &= \int_1^0 y^{1/2} (-1)(10-x) dy \\ &= (10-x) \int_0^1 y^{1/2} dy \\ &= (10-x) \frac{2}{3} y^{3/2} \Big|_0^1 \\ &= (10-x) \frac{2}{3} \end{aligned}$$

$$\begin{aligned} y &= 1 - \frac{t}{10-x} \\ \frac{dy}{dt} &= -\frac{1}{10-x} \\ dt &= -(10-x) dy \end{aligned}$$