MATH 4/50056 | Actuarial Mathematics II | Spring 2015
MTWR 8:50 | Dr. Kracht

Name: _____ .

Homework 1: Due Thursday, January 15, 2015

This is a little review from the first semester to help you clear those cobwebs out of your brain!

1. Suppose the lifetime distribution function for a turtle in the pond in my front yard is given by

$$F_0(t) = \begin{cases} 1 - \left(1 - \frac{t}{20}\right)^{1/4} & \text{for } 0 \leq t \leq 20 \\ 1 & \text{for } t \geq 20 \end{cases}$$

   (a) Find and simplify an expression for $S_0(t)$, the survival function for a newly hatched turtle.

   (b) Calculate the probability (rounded to two decimal places) that
      i. a newly hatched turtle survives beyond age 10

      ii. a turtle aged 8 dies before age 11

      iii. a turtle aged 5 survives beyond age 12

   (c) Derive and simplify an expression for the force of mortality, μ_x, for a turtle aged x.

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(d) Find and simplify an expression for ${}_tp_x$, the probability that a turtle aged x survives to at least age $x+t$.

(e) Find and simplify a formula for $\mathring{e}_x$, the complete expectation of life for a turtle aged x.

2. Recall that $e_{x:\overline{n}|} = E[\min(K_x, n)] = \sum_{k=1}^{n} {}_kp_x$. (See Exercise 2.16 in the text or Problem 4 on Homework 1 from last semester.)

   (a) Show that $e_x = e_{x:\overline{n}|} + {}_np_x \; e_{x+n}$.

   (b) You are given the following extract from a table with a 3-year select period.

   i.

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

   ii. $e_{64} = 5.10$

   Calculate $e_{[61]}$. *Hint: Use a select version of the formula above.*

3. For a whole life insurance of \$1000 on (50), you are given

   - The death benefit is payable at the end of the year of death.
   - Mortality follows the Illustrative Life Table (attached or posted on class web page).
   - $i = 0.05$ the first year, and $i = 0.06$ in subsequent years.

   Calculate the actuarial present value of this insurance.

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4. Consider the insurance benefit with the present value given by

$$Z = \begin{cases} 0 & \text{if } T_x \leq 5, \\ 100\, v^{T_x} & \text{if } 5 < T_x \leq 25, \\ 100\, v^{25} & \text{if } T_x > 25. \end{cases}$$

   (a) Write down a formula for the expected value for Z in integral or summation form.

   (b) Write down an expression in terms of standard actuarial functions for the expected value for Z.

5. Consider the following excerpt from a life table.

x	l_x	A_x
35	100,000.00	0.151375
36	99,737.15	0.158245
37	99,455.91	0.165386
38	99,154.72	0.172804
39	98,831.91	0.180505
40	98,485.68	0.188492

Compute each of the following given an effective interest rate of 6% per year.

(a) ${}_3E_{35}$

(b) ${}_{3|}A_{35}$

(c) $A^1_{35:\overline{3}|}$

(d) $\bar{A}_{35:\overline{3}|}$ assuming UDD

6. Bob, currently exact age 40, joined a defined benefit pension plan at exact age 35. His current salary is \$50,000 per year. He will retire at exact age 65.

   You are given:

   - Bob's salary will increase at the rate of 2% each year on his future birthdays.
   - The annual retirement benefit is 0.5% of the final three-year average salary for each year of service.
   - Bob wants to supplement this annual retirement benefit with an annuity, so that the total annual benefit is \$42,000.
   - Retirement benefits will commence at exact age 65 and are payable at the beginning of each year of life.
   - $\ddot{a}_{65} = 9.9$

   Calculate the amount Bob needs at age 65 to purchase the annuity to receive his desired annual retirement benefit.