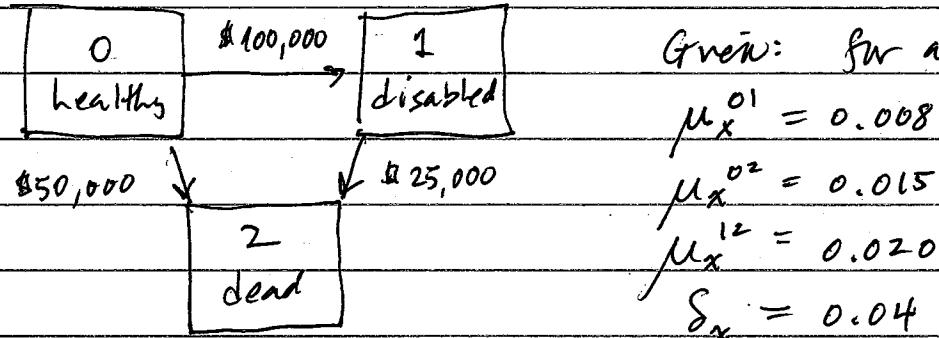


## Chapter 8 Exercise



First compute the relevant probabilities:

$$\begin{aligned}
 tP_{55}^{\bar{0}\bar{0}} &= tP_{55}^{00} = \exp \left\{ - \int_0^t (\mu_{55+s}^{01} + \mu_{55+s}^{02}) ds \right\} \\
 &= \exp \left\{ - \int_0^t (0.008 + 0.015) ds \right\} \\
 &= \exp \left\{ - \int_0^t 0.023 ds \right\} \\
 &= \exp \left\{ - 0.023s \Big|_0^t \right\} \\
 &= e^{-0.023t}
 \end{aligned}$$

$$\begin{aligned}
 tP_{55}^{\bar{1}\bar{1}} &= \exp \left\{ - \int_0^t \mu_{55+s}^{12} ds \right\} \\
 &= \exp \left\{ - \int_0^t 0.020 ds \right\} \\
 &= e^{-0.020t}
 \end{aligned}$$

$$tP_{55}^{01} = \int_0^t s P_{55}^{\bar{0}\bar{0}} \mu_{55+s}^{01} t-s P_{55+s}^{\bar{1}\bar{1}} ds$$

↙ The only way to start in state 0 and end in state 1 is to make exactly one transition

$$= \int_0^t e^{-0.023s} (0.008) e^{-0.020(t-s)} ds$$

$$= 0.008 e^{-0.020t} \int_0^t e^{-0.003s} ds$$

$$= 0.008 e^{-0.020t} \left( -\frac{1}{0.003} \right) \left( e^{-0.003t} - 1 \right)$$

$$= \frac{8}{3} e^{-0.020t} \left( 1 - e^{-0.003t} \right) = \frac{8}{3} \left( e^{-0.020t} - e^{-0.023t} \right).$$

EPV of benefit on transition from healthy to disabled:

$$\begin{aligned} \text{EPV} &= 100,000 \int_0^{20} e^{-st} t P_{55}^{00} \mu_{55+t}^{01} dt \\ &= 100,000 \int_0^{20} e^{-0.04t} e^{-0.023t} (0.008) dt \\ &= 100,000 \int_0^{20} 0.008 e^{-0.063t} dt \\ &= 800 \left[ \frac{1}{-0.063} (e^{-0.063t}) \right] \Big|_0^{20} \\ &= 800 \left( \frac{1}{-0.063} \right) [e^{-0.063(20)} - e^0] \\ &= 800 \left( \frac{1}{0.063} \right) (1 - e^{-1.26}) \end{aligned}$$

$$\approx \$9096.46$$

EPV of benefit on transition from healthy to dead:

$$\begin{aligned} \text{EPV} &= 50,000 \int_0^{20} e^{-st} t P_{55}^{00} \mu_{55+t}^{02} dt \\ &= 50,000 \int_0^{20} e^{-0.04t} e^{-0.023t} (0.015) dt \\ &= (50,000)(0.015) \int_0^{20} e^{-0.063t} dt \quad (\text{same integral as above}) \\ &\vdots \\ &= 750 \left( \frac{1}{0.063} \right) (1 - e^{-1.26}) \\ &= \$8527.93 \end{aligned}$$

EPV of benefit on transition from disabled to dead:

$$\begin{aligned} \text{EPV} &= 25,000 \int_0^{20} e^{-st} t P_{55}^{(0)} \mu_{55+t}^{12} dt \\ &= 25,000 \int_0^{20} e^{-0.04t} \left[ \frac{8}{3} (e^{-0.020t} - e^{-0.023t}) \right] (0.020) dt \\ &= \frac{(25,000)(8)(0.020)}{3} \int_0^{20} (e^{-0.060t} - e^{-0.063t}) dt \\ &= \frac{4000}{3} \left[ \frac{1}{0.06} (1 - e^{-0.06 \times 20}) - \frac{1}{0.063} (1 - e^{-0.063 \times 20}) \right] \\ &= \frac{4000}{3} \left[ \frac{1}{0.06} (1 - e^{-1.2}) - \frac{1}{0.063} (1 - e^{-1.26}) \right] \end{aligned}$$

$$\approx \$368.26$$

So the EPV of the total benefit is the sum of these:

$$9096.46 + 8527.93 + 368.26 = \underline{\underline{\$17992.65}}$$