1. \(f^{-1}(x) = x^5 + 2x^3 + 3x + 1\)
   (a) \(f^{-1}(1) = 7, f(1) = 0\)
   (b) The value \(x_0\) such that \(f(x_0) = 1\) is \(f^{-1}(1) = 7\).
   (c) The value \(y_0\) such that \(f^{-1}(y_0) = 1\) is \(f(1) = 0\).
   (d) The graph of \(f(x)\) is the graph of \(f^{-1}(x)\) reflected about the line \(y = x\).

2. Let \(f(x) = 3(2)^{-x} + 1\). Then \(f(x)\) is always decreasing, has a horizontal asymptote at \(y = 1\), and \(f(0) = 4\).

3. \(f(x) = 1.2(2)^{0.585x}\)

4. (a) \(f'(x) = 2\pi e^{2\pi x}\)
   (b) \(g'(x) = 2\pi e \cdot x^2 e^{-1}\)
   (c) \(h'(x) = 2\ln e \cdot (e^x)^{2x} = 2(1 + \ln e)(e^x)^{2x}\)
   (d) \(l'(x) = (\ln e) e^{(e^x)} \cdot 2e^{2x}\)

5. \(h'(x) = (1 + \sin \pi x)^{g(x)} \left[ g'(x) \ln(1 + \sin \pi x) + g(x) \frac{\pi \cos \pi x}{1 + \sin \pi x} \right];
\)
   \(h'(1) = 1^2 \cdot \left[ -1 \cdot 0 + 2 \pi \cdot (-1) \right] = -2\pi\)

6. (a) The slope is \((e^{-x^2})' = -2xe^{-x^2}\). If \(-2xe^{-x^2} = \frac{2}{e}\), then \(-x^2 = -1\) and \(-2x = 2\). So \(x = -1\) and the point is \((-1, \frac{1}{e})\).
   (b) The tangent line at \((-1, \frac{1}{e})\) is \(y = \frac{1}{e} + \frac{2}{e}(x + 1)\). Set \(y = 0\) and solve for \(x\) to get \(x = -\frac{3}{2}\).

7. (a) \(\lim_{x \to \infty} f(x) = \infty\)
   (b) \(f'(x) = \frac{1}{1 + x^2} \cdot 2x = \frac{2x}{1 + x^2}\). \(\lim_{x \to \infty} f'(x) = \lim_{x \to \infty} \frac{2x}{1 + x^2} = 0\)
   (c) The function increases at a slower and slower rate, but still goes to infinity.
CHAPTER 7 INVERSE FUNCTIONS

8. (a) ![Graph](image)

9. (a) Use \( f(1) = 1 \) and \( f(3) = 0 \) to get \( A = -\frac{1}{\ln 3} \) and \( B = 1 \).

   (b) Use \( f(0.2) = 0 \) to get \( K = 5 \).

10. (a) \( g'(x) = 5 \left( \frac{1}{5x} \right) = \frac{1}{x} \)

    (b) \( g(x) \) must differ from \( \ln x \) by a constant, so \( g(x) = \ln x + c \). Using \( g(1) = \int_1^5 \frac{dt}{t} = \ln 5 \), we get \( c = \ln 5 \), so \( g(x) = \ln x + \ln 5 \).

11. (a) \( e^{\ln x} = x \) is true only for positive values of \( x \) because \( \ln x \) is defined only for positive values of \( x \).

    This is because we cannot integrate across the discontinuity at \( t = 0 \).

    (b) \( \frac{1}{t} \) is positive for \( t > 0 \), so \( \ln x \) is increasing everywhere.

    (c) \( \ln \left( x^{1/2} \right) = \int_1^{x^{1/2}} \frac{1}{t} \, dt \). Let \( u = t^2 \). Then \( u^{1/2} = t \) so \( \frac{1}{2} u^{-1/2} \, du = dt \) and

    \[ \int_1^{x^{1/2}} \frac{1}{1} \, dt = \int_1^x 1 \cdot 2u \frac{1}{2} \, du = \frac{1}{2} \ln x. \]

    So \( \ln \left( x^{1/2} \right) = \frac{1}{2} \ln x \).

12. \( g'(x) = 2x \ln \left( \frac{1}{2} + |x| \right) = 0 \) where \( x = 0 \), \( x = \frac{1}{2} \), or \( x = -\frac{1}{2} \). \( g(x) \) has critical points at \( x = 0 \) and at \( x = \pm \frac{1}{2} \).

13. (a) The points of intersections are at \( x = 0 \) and \( x = c \), where \( c \approx 0.752 \).

    ![Graph](image)

    (b) \[ A = \int_{-1/2}^{1/2} \left| e^x - 1 - 2 \ln (x + 1) \right| \, dx \]

    \[ = \int_{-1/2}^{0} [e^x - 1 - 2 \ln (x + 1)] \, dx + \int_{0}^{c} [2 \ln (x + 1) - (e^x - 1)] \, dx \]

    \[ + \int_{c}^{2} [e^x - 1 - 2 \ln (x + 1)] \, dx \]

    where \( c \approx 0.752 \).