EXAM 1—Wednesday, January 29, 2003

Do the easier questions first. Read the directions carefully. GOOD LUCK!

1. (20 points) The function \( g \) is depicted below. Find each of the following (or state “does not exist”).

\[
\begin{align*}
\lim_{x \to -2^-} g(x) = \quad & \lim_{x \to -2^+} g(x) = \\
\lim_{x \to 1^-} g(x) = \quad & \lim_{x \to 1^+} g(x) = \\
\lim_{x \to 2^-} g(x) = \quad & \lim_{x \to 2^+} g(x) = \\
\lim_{x \to 4^-} g(x) = \quad & \lim_{x \to 4^+} g(x) = \\
g(-2) = \quad & g(1) = \quad & g(2) = \quad & g(4) = 
\end{align*}
\]

2. (10 points) Suppose that \( \lim_{x \to 10} f(x) = -5, \quad \lim_{x \to 10} g(x) = 20, \quad \text{and} \quad \lim_{x \to 10} h(x) = \frac{1}{2}. \)

Compute each limit using the limit laws. Write out each step completely showing how you are using the limit laws. (You need not label the steps with the number of the law.) If a limit does not exist or cannot be computed with the information given, so state and explain why.

(a) \( \lim_{x \to 10} (4f(x) - 2g(x) + 6) \)

(b) \( \lim_{x \to 10} \frac{f(x)g(x)}{h(x)} \)
3. (20 points) Find the limit algebraically, if it exists. Show all work. If the limit does not exist, explain why.

(a) \( \lim_{x \to 0} \frac{x^2 - 64}{x + 8} \)

(b) \( \lim_{x \to -8} \frac{x^2 - 64}{x + 8} \)

(c) \( \lim_{t \to 5} \frac{\frac{1}{t} - \frac{1}{5}}{x - 5} \)

(d) \( \lim_{x \to -2} \frac{x - 2}{|x - 2|} \)
4. (15 points)

(a) Complete the statement of the formal definition of **continuity**.

A function \( f \) is continuous at a number \( a \) provided

(b) Let \( f(x) = \begin{cases} 
2x + 1 & \text{for } x < 3 \\
10 - x & \text{for } x \geq 3 
\end{cases} \).

Use the definition of continuity to prove that \( f \) is continuous at \( x = 3 \).

5. (15 points) Use the Intermediate Value Theorem (IVT) to prove that the function \( f(x) = 4x^5 + x^2 - 3 \) has a root (\( x \)-intercept) on the interval \((0, 1)\). (\textit{Hint: Verify all hypotheses of IVT.})
6. (20 points) Let \( f(x) = \sqrt{x} \).

(a) Find an expression for the slope of the secant line through points \( P(9, f(9)) \) and \( Q(x, f(x)) \).

(b) Find the slope of the tangent line to the curve \( y = f(x) \) at the point \( P(9, 3) \). (Use limits to find the exact value.)

(c) Find the equation of this tangent line, writing it in the form “\( y = mx + b \).”

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**Bonus. (10 points) (Do on another sheet of paper.)**

Let \( f(x) = \frac{1}{x} \). Then the point \((4, 0.25)\) lies on the graph of \( f \). Find a number \( \delta \) such that

\[
\left| \frac{1}{x} - 0.25 \right| < 0.05 \quad \text{whenever} \quad |x - 4| < \delta
\]

Show that your answer is correct. Sketch a diagram illustrating the situation.