1. (16 points) The function $g$ is depicted below. Find each of the following (or state “does not exist”).

$$
\lim_{x \to -2^-} g(x) = \quad \lim_{x \to 1^-} g(x) = \quad \lim_{x \to -2^+} g(x) = \quad \lim_{x \to 4^-} g(x) = \\
\lim_{x \to -2^+} g(x) = \quad \lim_{x \to 1^+} g(x) = \quad \lim_{x \to -2^+} g(x) = \quad \lim_{x \to 4^+} g(x) = \\
\lim_{x \to 2} g(x) = \quad \lim_{x \to 1} g(x) = \quad \lim_{x \to 2} g(x) = \quad \lim_{x \to 4} g(x) = \\
g(-2) = \quad g(1) = \quad g(2) = \quad g(4) = 
$$

2. (10 points) Find the limit algebraically, if it exists. If the limit does not exist, explain why not. Show all work.

$$
\lim_{x \to 5} \frac{\sqrt{x - 1} - 2}{x - 5}
$$
3. (10 points) Find the limit algebraically, if it exists. If the limit does not exist, explain why not. Show all work.

\[ \lim_{x \to -7^-} \frac{2x + 14}{|x + 7|} \]

4. (10 points) Answer each question yes or no. If yes, explain why (naming any theorems you are using). If no, draw a counterexample.

(a) Suppose \( f \) is continuous on the interval \([3, 10]\) and \( f(3) = 5 \) and \( f(10) = 1 \). Must there exist some number \( c \) in the interval \((3, 10)\) such that \( f(c) = 2 \)?

Circle one: **Yes** or **No** because

(b) Suppose \( f \) is continuous on the interval \([3, 10]\) and \( f(3) = 5 \) and \( f(10) = 1 \). Must there exist some number \( c \) in the interval \((3, 10)\) such that \( f(c) = 2 \)?

Circle one: **Yes** or **No** because
5. (15 points)

(a) Complete the statement of the formal definition of continuity.

A function $f$ is continuous at a number $a$ provided

(b) Let $f(x) = \begin{cases} 
  x^2 + 1 & \text{for } x < 5 \\
  10x - 24 & \text{for } x \geq 5 
\end{cases}$.

Use the definition of continuity to prove that $f$ is continuous at $x = 5$. 

6. (20 points) Let \( f(x) = \frac{1}{x} \).

   (a) Find the \( y \)-coordinate of the point on the curve \( y = f(x) \) where \( x = 2 \).

   (b) Find an expression for the slope of the secant line through points \( P(2, f(2)) \) and \( Q(x, f(x)) \).

   (c) Use the (limit) definition to find the slope of the tangent line to the curve \( y = f(x) \) at the point where \( x = 2 \). *Show all steps.*

   (d) Find the equation of this tangent line, writing it in the form \( y = mx + b \).
7. (10 points) Consider the function $f$ whose graph is given. Define a new function $g$ by $g(x) = f(-x) + 3$.

(a) List a sequence of transformations from $f$ to $g$.

(b) Use this to sketch the graph of the function $g$. Label at least 5 points with their ordered pairs.

8. (10 points) Find exact values (in radians) of all solutions to the given equation. Show your work.

$$\cos x = 2 \sin x \cos x$$
**Bonus A.** (10 points) Use the Intermediate Value Theorem (IVT) to prove that the equation \( \sin x = 2 \cos x \) has a root (solution) on the interval \( \left(0, \frac{\pi}{2}\right)\). *(Hint: Verify all hypotheses of IVT.)*

**Bonus B.** (5 points) Does there exist a function whose domain is the set of all real numbers but which is discontinuous at every real number?

*Circle one:* Yes or No

If yes, give an example. If no, explain why not.