1. (10 points) Find the exact value of each expression or else indicate “does not exist.”
   (a) \( \log_4 8 \)
   (b) \( e^{2 \ln 5} \)
   (c) \( \sin^{-1} \left( -\frac{1}{2} \right) \)
   (d) \( \tan^{-1} (1) \)
   (e) \( \sin^{-1} \left( \sin \left( \frac{2\pi}{3} \right) \right) \)

2. (10 points) Let \( y = \ln x \). Show that \( \frac{dy}{dx} = \frac{1}{x} \) by using the fact that \( \frac{d}{dx} e^x = e^x \).

3. (10 points) Evaluate each limit. Show your reasoning.
   (a) \( \lim_{x \to 0} \frac{e^x - 1}{\sin x} \)
   (b) \( \lim_{x \to 0^+} \sqrt{x} \ln x \)
4. (10 points)

(a) Complete the statement of the Mean Value Theorem.

Suppose \( f \) is a function satisfying the following hypotheses.

\( i. \) \( f \) is continuous on the closed interval \([a, b]\);

\( ii. \) \( f \) is differentiable on the open interval \((a, b)\);

Then

(b) Explain (in complete sentences) what the Mean Value Theorem means geometrically. You might also wish to illustrate with a sketch.

5. (10 points) Use calculus to find the absolute minimum and absolute maximum values of the function

\( f(x) = x^3 - 3x + 1 \) on the interval \([-2, 3]\). Give exact values. Show your reasoning clearly.

The absolute minimum value of \( f \) on \([-2, 3]\) is ________ and it occurs at \( x = \) ________________.

The absolute maximum value of \( f \) on \([-2, 3]\) is ________ and it occurs at \( x = \) ________________.
6. (20 points) Let \( f(x) = \frac{x^2 + x + 1}{x} = x + 1 + \frac{1}{x} \).

(a) Give the equations for all vertical asymptotes of the graph of \( f \).

(b) Give the equation for the slant asymptote of the graph of \( f \).

(c) Verify that your slant asymptote is correct. *You may show just one limit.*

(d) Use the fact that \( f'(x) = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2} \) to find all critical numbers of \( f \) and to make a sign chart showing all intervals of increase/decrease of \( f \).

(e) Use the fact that \( f''(x) = -\frac{2}{x^3} \) to find all possible inflection points of \( f \) and to make a sign chart showing all intervals of concavity of the graph of \( f \).
7. (20 points) Let \( f \) be continuous on \( \mathbb{R} \) with \( f'(-1) \) undefined, \( f'(0) = 0 \), \( f'(2) \) undefined, and \( f''(0) = 0 \).

(a) Fill in the blanks in the sign charts with the terms “increasing,” “decreasing,” “concave up,” or “concave down.”

\[
\begin{array}{c|cccc}
\text{interval} & (-\infty,-1) & (-1,0) & (0,2) & (2,\infty) \\
\hline
\text{sign of } f'(x) & - & - & - & + \\
\text{behavior of } f & \\
\end{array}
\]

\[
\begin{array}{c|cccc}
\text{interval} & (-\infty,-1) & (-1,0) & (0,2) & (2,\infty) \\
\hline
\text{sign of } f''(x) & - & + & - & - \\
\text{behavior of } f & \\
\end{array}
\]

(b) Give the \( x \)-coordinates of all of the following (or indicate “none”).

i. critical points of \( f \): \( x = \) ___________

ii. inflection points of \( f \): \( x = \) ___________

iii. local maximum points of \( f \): \( x = \) ___________

iv. local minimum points of \( f \): \( x = \) ___________

(c) Sketch a possible graph of \( f \) labeling all important points (with ordered pairs) and indicating all behavior clearly.
EXAM 3—Take-Home Question
Due Wednesday, November 2, 2005 at the beginning of class

8. (15 points) Prove that the equation \( x^5 + 3x - 2 = 0 \) has exactly one real solution.

Give a formal mathematical proof. Use complete sentences. Explain your reasoning clearly. In particular, be explicit about which theorems you are using and how you are using them.

Proof.