1. (20 points) The graph of the function \( f \) is shown below.

![Graph of f(t)](image)

(a) Evaluate each of the following definite integrals.

i. \[ \int_{-2}^{1} f(t) \, dt = \] 

ii. \[ \int_{0}^{-3} f(t) \, dt = \] 

iii. \[ \int_{6}^{3} f(t) \, dt = \] 

iv. \[ \int_{5}^{5} f(t) \, dt = \]

(b) Define the function \( A \) by \( A(x) = \int_{0}^{x} f(t) \, dt \). Find each of the following.

i. \( A(2) = \) 

ii. \( A'(2) = \) 

iii. \( A(3) = \) 

iv. \( A'(3) = \)

(c) Suppose the function depicted above represents the velocity \( v(t) \) (in miles per hour) of a moving object at time \( t \) hours.

i. Find the displacement of the object from \( t = 0 \) to \( t = 5 \) seconds.

ii. Find the total distance traveled (without regard to direction) by the object from \( t = 0 \) to \( t = 5 \) seconds.
2. (15 points) Consider the function \( f(x) = 9 - x^2 \). We wish to approximate the integral \( \int_{-3}^{3} (9 - x^2) \, dx \) using the right-hand sum \( R_4 \).

(a) Carefully sketch the graph of \( f \). Then sketch the rectangles for \( R_4 \).

(b) Find the length, \( \Delta x \), of each subinterval.

(c) Fill in the table (including the rest of the second row of labels). Give exact answers written as fractions.

<table>
<thead>
<tr>
<th>sample point for ( i^{th} ) subinterval</th>
<th>height of ( i^{th} ) rectangle</th>
<th>area of ( i^{th} ) rectangle</th>
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<tbody>
<tr>
<td>( i ) ( x_i^* )</td>
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(d) Find \( R_4 \). First give the symbolic form and then find the exact answer written as a fraction reduced to lowest terms.
3. (10 points)

(a) Express as a definite integral. (Do not evaluate.)
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\ln(x_i^n - 2)}{x_i^n} \Delta x \quad \text{on } [3, 5].
\]

(b) Evaluate. \[ \int_{-1}^{1} e^{3x} \, dx \]

(c) Given that \[ \int_{0}^{5} f(t) \, dt = \frac{3}{4} \], find \[ \int_{5}^{0} f(x) \, dx \].

(d) If \[ \int_{10}^{11} f(x) \, dx = -5 \] and \[ \int_{10}^{11} g(x) \, dx = 3 \], find \[ \int_{10}^{11} (f(x) - 2g(x)) \, dx \]

(e) If \[ 4 \leq f(x) \leq 7 \] for all \[ x \in [-1, 5] \], find a lower bound and an upper bound for the value of \[ \int_{-1}^{5} f(x) \, dx \].
4. (5 points) Find the most general antiderivative $F$ for the following function. Show your reasoning clearly. Check your answer by differentiating.

$$f(x) = e^{4 \ln(x) + \ln(2x+1)}$$

Check:

5. (10 points) If $f''(x) = \sin x \cos x$ and $f(0) = 2$ and $f(\pi) = -1$, find $f(x)$.

6. (5 points) Find $\frac{d}{dx} \int_{1}^{x} \sin^2 t \, dt$. 
7. (10 points) For $x \in [0, \infty)$, define the function $A$ by $A(x) = \int_0^x (5t + 2) \, dt$.

(a) Find $A'(x)$.

(b) Find an algebraic expression for $A(x)$. Use any method, but show your reasoning clearly.

8. (10 points) Evaluate each definite integral. Give exact answers. If the answer is a rational number, express it as a fraction in lowest terms.

(a) $\int_{-2}^1 (x^3 + 2) \, dx$

(b) $\int_{\pi/6}^{\pi} \sin x \, dx$
9. (25 points) A Norman window has the shape of a rectangle surmounted by a semicircle (whose diameter is equal to the width of the window). If the perimeter of the window is 20 ft, find the dimensions of the window so that the greatest possible amount of light is admitted. That is, find the dimensions that will maximize the area of the window.

Show your reasoning. Be sure to:

- Introduce all variables with “Let” statements. Include the units.
- Draw and label at least two diagrams showing different possible fields.
- Set up a function to be optimized and constraint equations.
- Use calculus to find the maximum or minimum point.
- Verify that you have indeed found the maximum or minimum point (on the appropriate domain).
- Answer the question posed in the problem in a complete sentence, using appropriate units.