1. (20 points) The function $g$ is depicted below.

(a) Find each of the following (or state “does not exist”).

$$\lim_{x \to -2^-} g(x) = \quad \lim_{x \to -2^+} g(x) = \quad \lim_{x \to 1^-} g(x) = \quad \lim_{x \to 2^-} g(x) = \quad \lim_{x \to 4^-} g(x) = \quad \lim_{x \to 4^+} g(x) =$$

$$g(-2) = \quad g(1) = \quad g(2) = \quad g(4) =$$

(b) Circle Yes or No for each of the following.

i. Is $g$ continuous at $-2$?  Yes  No

ii. Is $g$ continuous at $1$?  Yes  No

iii. Is $g$ continuous at $2$?  Yes  No

iv. Is $g$ continuous at $4$?  Yes  No
2. (20 points) Find each limit algebraically, if it exists or is infinite. If the limit does not exist, explain why not. Show all work.

(a) \( \lim_{w \to 3} \sqrt[3]{1 - w^2} \)

(b) \( \lim_{x \to 1} \frac{\sqrt{3x - 2} - 1}{x - 1} \)

(c) \( \lim_{x \to 10} \frac{x - 10}{x - 10} \)

(d) \( \lim_{x \to 5^+} \left( \frac{1}{|5 - x|} + \frac{1}{5 - x} \right) \)
3. (15 points)

(a) Complete the statement of the formal definition of continuity.

A function $f$ is continuous at a number $a$ provided

(b) Let $f(x) = \begin{cases} 
1 - x & \text{for } x < 1 \\
\sqrt{x - 1} & \text{for } x \geq 1 
\end{cases}$.

Use the definition of continuity to prove that $f$ is continuous at $x = 1$.

4. (15 points) Use the Intermediate Value Theorem (IVT) to prove that the equation $\sin x = 1 - x$ has a root (solution) on the interval $(0, \pi)$. (Hint: Verify all hypotheses of IVT.)
5. (15 points) An arrow is shot straight up on the moon at a velocity of 58 meters per second. Then its height \( h \) (in meters) above the ground at \( t \) seconds is given by
\[
h(t) = 58t - 0.83t^2.
\]
(a) Find the average velocity, \( v_{\text{avg}} \), of the arrow over the given time intervals. Show your reasoning. Round your answers to two decimal places.

i. \([0.9, 1]\)

ii. \([1, 1.1]\)

(b) Explain how average velocity is used to find the instantaneous velocity of the arrow at \( t = 1 \) second exactly.

6. (15 points) Suppose the tangent line to \( y = f(x) \) at the point \((2, -5)\) passes through the point \((1, 4)\).

(a) Find \( f(2) \).

(b) Find \( f'(2) \). Show your reasoning.
7. (15 points) Find the derivative $f'(x)$ using the definition of derivative where $f(x) = \frac{2}{3x}$.