EXAM 4—Friday, March 24, 2006

Do the easier questions first. Read the directions carefully. GOOD LUCK!

1. (16 points) The graph of the function \( f \) is shown below.

(a) Evaluate each of the following definite integrals.

i. \( \int_{-2}^{1} f(t) \, dt = \) 

ii. \( \int_{0}^{-3} f(t) \, dt = \) 

iii. \( \int_{6}^{3} f(t) \, dt = \) 

iv. \( \int_{5}^{5} f(t) \, dt = \)

(b) Define the function \( A \) by \( A(x) = \int_{0}^{x} f(t) \, dt \). Find each of the following.

i. \( A(2) = \) 

ii. \( A'(2) = \) 

iii. \( A(3) = \) 

iv. \( A'(3) = \)

2. (4 points) Suppose an object is moving with velocity \( v(t) \) miles per hour at time \( t \) hours.

(a) Write an integral that represents the displacement of the object from \( t = 0 \) to \( t = 5 \) hours.

(b) Write an integral that represents the total distance traveled (without regard to direction) by the object from \( t = 0 \) to \( t = 5 \) hours.
3. (20 points) Consider the function \( f(x) = x^2 + 1 \). We wish to approximate the area under the curve \( y = f(x) \) from \( x = -2 \) to \( x = 1 \) using the right-hand sum \( R_6 \).

(a) Carefully sketch the graph of \( f \). Also sketch the rectangles for \( R_6 \). You might wish to fill in the table below first.

(b) Find the length, \( \Delta x \), of each subinterval. Write out the computation.

(c) Fill in the table. The rest of the second row of labels should be given in general (symbolic) form. The entries in the remaining rows should be numerical values. Give exact values written as fractions.

<table>
<thead>
<tr>
<th>( i )</th>
<th>sample point for ( i^{th} ) subinterval</th>
<th>height of ( i^{th} ) rectangle</th>
<th>area of ( i^{th} ) rectangle</th>
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<tbody>
<tr>
<td>1</td>
<td>( x_1^* )</td>
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(d) Find \( R_6 \). First give the symbolic form using sigma (summation) notation and then find the exact answer written as a fraction reduced to lowest terms. HINT: Use the results of the computations in your table above.
4. (10 points) An object is moving so that
\[ a(t) = 10 \sin t + 3 \cos t \quad \text{and} \quad s(0) = 0 \quad \text{and} \quad s(2\pi) = 12\pi \]
where \(a(t)\) denotes acceleration (in m/s\(^2\)) and \(s(t)\) denotes position (in m) at time \(t\) seconds. Find a formula for \(s(t)\). Show your reasoning.

5. (10 points)

(a) Express as a definite integral. (Do not evaluate.)
\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\sqrt{x_i^3 - 2}}{(x_i^*)^2 + 10} \Delta x \quad \text{on } [3, 5].
\]

(b) Evaluate.
\[
\int_{\pi}^{\pi} \cos^2 \theta \, d\theta =
\]

(c) If \(\int_{2}^{4} f(t) \, dt = -\frac{1}{9}\), then \(\int_{2}^{4} f(x) \, dx =
\]

(d) If \(\int_{10}^{11} f(x) \, dx = -4\) and \(\int_{10}^{11} g(x) \, dx = 5\), then \(\int_{10}^{11} (3f(x) - 2g(x)) \, dx =
\]

(e) If \(3 \leq f(x) \leq 10\) for all \(x \in [-1, 5]\), find a lower bound and an upper bound for the value of \(\int_{-1}^{5} f(x) \, dx\).

Express as a compound inequality.
6. (10 points) For $x \in [1, \infty)$, define the function $A$ by $A(x) = \int_{1}^{x} 2t \, dt$.

(a) Find $A'(x)$.

(b) Find an algebraic expression for $A(x)$. Use any method, but show your reasoning clearly.

7. (5 points) Find $\frac{d}{dx} \int_{1}^{x^5} \sin t \cos t \, dt$.

8. (10 points) Evaluate each definite integral. Show all work. Give exact answers.

(a) $\int_{-1}^{2} (3x^2 + 5) \, dx$

(b) $\int_{0}^{\pi/2} \cos \theta \, d\theta$
9. (25 points) An ostrich rancher wishes to enclose a rectangular pen and then divide it in half with a fence parallel to one of the sides of the rectangle so that the girl ostriches can be separated from the boy ostriches. The total enclosed area must be 9600 square feet. Find the dimensions which minimize the amount of fence needed.

*Show your reasoning. Be sure to:*

- Introduce all variables and functions with “Let” statements. Include the units.
- Draw and label two possible diagrams.
- Verify that you have indeed found the maximum or minimum point (on the appropriate domain).
- Answer the question posed in the problem in a complete sentence, using appropriate units.