Final Exam Review II

1. Give all possible meanings of each mathematical expression from the list below. You may use each letter any number of times.

<table>
<thead>
<tr>
<th>Mathematical Expression</th>
<th>Meaning(s) (list by letter)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(a) )</td>
<td>( b, f )</td>
</tr>
<tr>
<td>( \frac{f(a+h) - f(a)}{h} )</td>
<td>( c, d )</td>
</tr>
<tr>
<td>( \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} )</td>
<td>( a, e )</td>
</tr>
<tr>
<td>( f'(a) )</td>
<td>( a, e )</td>
</tr>
</tbody>
</table>

Possible Meanings:

(a) the slope of a tangent line to the graph \( y = f(x) \)
(b) the \( y \)-coordinate of a point on the graph \( y = f(x) \)
(c) the average rate of change of \( f \) with respect to \( x \)
(d) the slope of a secant line to the graph \( y = f(x) \)
(e) the instantaneous rate of change of \( f \) with respect to \( x \)
(f) an output of the function \( f \)

2. Let \( f \) be a differentiable function. Fill in the table with a mathematical expression corresponding to each description.

<table>
<thead>
<tr>
<th>Description</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>output of the function ( f ) when the input is 4</td>
<td>( f(4) )</td>
</tr>
<tr>
<td>the average rate of change of ( f ) with respect to ( x ) between ( x = 4 ) and ( x = 4.5 )</td>
<td>( \frac{f(4.5) - f(4)}{0.5} )</td>
</tr>
<tr>
<td>the instantaneous rate of change of ( f ) with respect to ( x ) at ( x = 4 )</td>
<td>( \lim_{h \to 0} \frac{f(4+h) - f(4)}{h} )</td>
</tr>
<tr>
<td>the instantaneous rate of change of ( f ) with respect to ( x ) at ( x = 4 ) (another expression)</td>
<td>( f'(4) )</td>
</tr>
</tbody>
</table>
3. Let \( f \) be a differentiable function. Fill in the table with a mathematical expression or equation corresponding to each description.

<table>
<thead>
<tr>
<th>Description</th>
<th>Mathematical Expression or Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-coordinate of the point on the graph ( y = f(x) ) where ( x = 27 )</td>
<td>( f(27) )</td>
</tr>
<tr>
<td>height above the ( x )-axis of the point on the graph ( y = f(x) ) where ( x = 27 )</td>
<td>( f(27) )</td>
</tr>
<tr>
<td>slope of the secant line through the points on the graph ( y = f(x) ) where ( x = 27 ) and where ( x = 30 )</td>
<td>( \frac{f(30) - f(27)}{3} )</td>
</tr>
<tr>
<td>slope of the tangent line to the graph ( y = f(x) ) at the point where ( x = 27 )</td>
<td>( \lim_{h \to 0} \frac{f(27 + h) - f(27)}{h} )</td>
</tr>
<tr>
<td>slope of the tangent line to the graph ( y = f(x) ) at the point where ( x = 27 ) (another expression)</td>
<td>( f'(27) )</td>
</tr>
<tr>
<td>equation to find the ( x )-coordinates of all points on the graph ( y = f(x) ) whose ( y )-coordinates are 32</td>
<td>( f(x) = 32 )</td>
</tr>
<tr>
<td>equation to find the ( x )-coordinates of all points at which the line tangent to the curve ( y = f(x) ) has slope 32</td>
<td>( f'(x) = 32 )</td>
</tr>
<tr>
<td>equation to find the ( x )-coordinates of all points at which the ( y )-coordinate is equal to the slope of the line tangent to the curve ( y = f(x) )</td>
<td>( f(x) = f'(x) )</td>
</tr>
</tbody>
</table>

4. Let \( C(x) \) be the cost, in dollars, of manufacturing \( x \) widgets. Fill in the table with a mathematical expression and appropriate units corresponding to each description.

<table>
<thead>
<tr>
<th>Description</th>
<th>Mathematical Expression</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost of manufacturing 100 widgets</td>
<td>( C(100) )</td>
<td>$</td>
</tr>
<tr>
<td>cost of manufacturing the 101\textsuperscript{st} widget</td>
<td>( C(101) - C(100) )</td>
<td>$</td>
</tr>
<tr>
<td>average rate of change of cost from a production level of 100 widgets to a production level of 101 widgets</td>
<td>( \frac{C(101) - C(100)}{101 - 100} )</td>
<td>$/widget</td>
</tr>
<tr>
<td>instantaneous rate of change of cost at a production level of 100 widgets</td>
<td>( \lim_{h \to 0} \frac{C(100 + h) - C(100)}{h} )</td>
<td>$/widget</td>
</tr>
<tr>
<td>instantaneous rate of change of cost at a production level of 100 widgets (another expression)</td>
<td>( C'(100) )</td>
<td>$/widget</td>
</tr>
</tbody>
</table>
5. Let $x$ and $f(x)$ represent the given quantities. Fix $x = a$ and let $h$ be a small positive number. Give an interpretation of the quantities

1. \[ \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \] and \[ \frac{f(a+h) - f(a)}{h} \] where

(a) $x$ denotes time and $f(x)$ denotes the population of black squirrels at time $x$.

1. represents the average rate of growth of squirrel population.

2. represents the instantaneous rate of growth of squirrel population.

(b) $x$ denotes time (in hours) and $f(x)$ denotes distance from Kent (in miles).

1. represents the average velocity in \( \text{mi/hr} \).

2. represents instantaneous velocity in \( \text{mi/hr} \).

(c) $x$ denotes altitude and $f(x)$ denotes atmospheric pressure.

1. represents average rate of change of atmospheric pressure w.r.t. altitude.

2. represents instantaneous rate of change of atmospheric pressure w.r.t. altitude.

(d) $x$ denotes the speed of a car (in mph) and $f(x)$ denotes the fuel economy of the car measured in miles per gallon (mpg).

1. represents average rate of change of fuel economy w.r.t. speed in \( \text{miles per gallon/mile per hour} \).

2. represents instantaneous rate of change of fuel economy w.r.t. speed in \( \text{miles per gallon/mile per hr} \).

6. Let $T(t)$ be temperature, in degrees Fahrenheit, at time $t$. For each scenario, fill in the banks with one of the symbols $<$, $>$, $=$, or $?$ (if the sign cannot be determined).

(a) The temperature held steady at 70° all afternoon.

\[ T(t) \geq 0 \quad T'(t) = 0 \quad T''(t) = 0 \]

(b) The temperature increased from the low in the 60’s at a slow but steady rate.

\[ T(t) \geq 0 \quad T'(t) \geq 0 \quad T''(t) = 0 \]

(c) At midnight, the temperature was 0° and it has been falling more and more rapidly ever since.

\[ T(t) \leq 0 \quad T''(t) \leq 0 \quad T'''(t) \leq 0 \]

(d) As the sun came out, the temperature increased more and more quickly.

\[ T(t) ? 0 \quad T'(t) \geq 0 \quad T''(t) \geq 0 \]

(e) The temperature is still falling, although not as rapidly as earlier in the evening.

\[ T(t) ? 0 \quad T'(t) < 0 \quad T''(t) \geq 0 \]
7. (a) If \( w'(t) \) is the rate of growth of a child in pounds per year, what does \( \int_{5}^{10} w'(t) \, dt \) represent?
   
   *This represents the amount of weight, in pounds, the child gains from age 5 to age 10.*

(b) If a honeybee population starts with 100 bees and increases at a rate \( n'(t) \) bees per week, what does \( 100 + \int_{0}^{15} n'(t) \, dt \) represent?

   *This represents the size of the population of honeybees at 15 weeks.*

(c) If oil leaks from a tank at a rate of \( r'(t) \) gallons per minute, what does \( \int_{0}^{120} r'(t) \, dt \) represent?

   *This represents the total amount of oil, in gallons, that has leaked in the first 120 minutes.*

8. For each of the following, give the full definition, in complete sentences.

(a) State the definition of limit.

   *See p. 25*

(b) State the definition of continuity (at a point).

   *See p. 46*

(c) State the definition of derivative.

   *See pp. 77-78*
(d) State the definition of critical number.

see p. 146

(e) State the definition of the definite integral.

see p. 206

9. (a) State the Squeeze Theorem.

see p. 41

(b) Interpret the Squeeze Theorem geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.
10. (a) State the Intermediate Value Theorem.

   \[ \text{see p. 52} \]

(b) Interpret the Intermediate Value Theorem geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.

   \[ \text{see p.} \]

11. State the theorem which gives the relationship between continuity and differentiability.

   \[ \text{see p. 58} \]

12. State the Extreme Value Theorem.

   \[ \text{p. 143} \]


   \[ \text{p. 144} \]
14. (a) State Rolle’s Theorem.

(b) Interpret Rolle’s Theorem geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.

15. (a) State the Mean Value Theorem.

(b) Interpret the Mean Value Theorem geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.
16. (a) State the Fundamental Theorem of Calculus (Part I).

\[ \text{Evaluation Thm p. 218} \]
\[ \text{or Net Change Thm p. 222} \]
\[ \text{or FTC (Part I) p. 229} \]

\(\text{also FTC}\)

(b) Interpret the Fundamental Theorem of Calculus (Part I) geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.

17. (a) State the Mean Value Theorem for Integrals.

\(\text{p. 233}\)

(b) Interpret the Mean Value Theorem for Integrals geometrically. (Explain what it means in plain English.) Use complete sentences supplemented by diagrams.