Permutations: The Maximum Number of Columns in a Preference Schedule

Each preference ballot is a permutation or ordering of all of the candidates in the election. Each column in a preference schedule represents one such ordering. So the maximum number of columns possible in a preference schedule is the number of distinct orderings of the candidates. This depends only on the number of candidates.

Example: Two candidates: Suppose there are 2 candidates, say A and B. Then there are just 2 ways to order them: (A, B) and (B, A). So there are just two columns possible in the preference schedule in an election with only two candidates. ■

Example: Three candidates: Suppose there are 3 candidates, say A, B, and C. Let us systematically list the possible orderings:

(A, B, C)	(B, A, C)	(C, A, B)
(A, C, B)	(B, C, A)	(C, B, A).

The first column gives all permutations beginning with A, the second column gives all permutations beginning with B, and the third column gives all permutations beginning with C. Once we choose the first element in the ordering, we have 2 choices for the second, and after that choice is made, only one choice for the last candidate in the ordering. This observation is how we will count the number of possible orderings when there are more candidates. ■

<u>Example</u>: Four candidates: Suppose there are 4 candidates, say A, B, C, and D. Let us try to count the number of permutations (orderings) of these 4 candidates without writing down all of the permutations. We can imagine having a box for each position in the ordering and we can move from left to right, placing a candidate in each box. Once a candidate is placed in position, we can't place her in another position. (So we cannot have (A, A, B, C), for example.)

$$(\square, \square, \square, \square)$$

Choices: A, B, C, D

↑ 1st position ↑ 2nd ↑ 3rd ↑ 4th

<u>1st position</u>: We have 4 choices: A, B, C, or D.

<u>2nd position</u>: We have just 3 choices – whichever 3 remain after the 1st position is chosen.

<u>3rd position</u>: There are just 2 choices remaining.

<u>4th position</u>: We have only 1 choice left.

To find the total number of permutations, we use the Multiplication Principle: take the product of the number of choices for each position: $4 \cdot 3 \cdot 2 \cdot 1 = 24$. So there are 24 possible columns in a preference schedule when there are 4 candidates. ■