## Who is the Girl Scout Leader of the Year?

The twenty-nine girls in Girl Scout troop 224 are voting on which of their four leaders, Miss Darci, Miss Misty, Miss Michelle, or Miss Pam, should be Leader of the Year. Here is the preference schedule for the election.

| \# voters | $\mathbf{1 1}$ | $\mathbf{7}$ | $\mathbf{7}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st place | Pam | Michelle | Darci | Misty | Michelle |
| 2nd place | Misty | Misty | Michelle | Darci | Darci |
| 3rd place | Michelle | Darci | Misty | Michelle | Misty |
| 4th place | Darci | Pam | Pam | Pam | Pam |

We will find the winner of Leader of the Year using four different democratic voting methods.

## The Plurality Method

In this method, we count the number of first-place votes each candidate receives. The candidate with the highest number of first-place votes is declared the winner of the election. Compute the number of first-place votes for each candidate:

Miss Pam:

Miss Misty:

Miss Michelle:

Miss Darci:

Who is the winner of Leader of the Year using the Plurality Method?

## The Borda Count Method

In this method, points are assigned for each vote. Since there are four candidates, a candidate earns 4 points for each first-place vote, 3 points for each second-place vote, 2 points for each third-place vote, and 1 point for each last-place vote. The candidate with the highest number of Borda points is declared the winner of the election. Compute the Borda Count for each candidate:

Miss Pam: $(11 \times 4)+(7 \times 1)+(7 \times 1)+(3 \times 1)+(1 \times 1)=44+7+7+3+1=62$
Miss Misty: $(11 \times \quad)+(7 \times \quad)+(7 \times \quad)+(3 \times \quad)+(1 \times \quad)=$

Miss Michelle:

Miss Darci:

Who is the winner of Leader of the Year using the Borda Count Method?

| \# voters | $\mathbf{1 1}$ | $\mathbf{7}$ | $\mathbf{7}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st place | Pam | Michelle | Darci | Misty | Michelle |
| 2nd place | Misty | Misty | Michelle | Darci | Darci |
| 3rd place | Michelle | Darci | Misty | Michelle | Misty |
| 4th place | Darci | Pam | Pam | Pam | Pam |

## The Pairwise Comparison Method

In this method, we compare each candidate with each other candidate in a head-to-head competition. The candidate who wins the most head-to-head competitions is declared the winner of the election.

| Pam | Misty |
| :---: | :---: |
| 11 | 7 |
|  | 7 |
|  | 3 |
|  | 1 |
| $\mathbf{1 1}$ | $\mathbf{1 8}$ |



| Pam | Michelle |
| :---: | :---: |
| 11 | 7 |
|  | 7 |
|  | 3 |
|  | 1 |
| $\mathbf{1 1}$ | $\mathbf{1 8}$ |




Each candidate earns one point for each pairwise comparison she wins and a half-point for each tie. Compute the number of points earned by each candidate:
Pam: Misty: Michelle: Darci:

Who is the winner of Leader of the Year using the Pairwise Comparison Method?

## The Plurality with Elimination Method

In each Round, we first check to see if there is a majority candidate (a candidate with more than half the first-place votes). If there is, we declare that candidate the winner. If not, we eliminate the candidate with the least number of first-place votes and repeat the process.
How many votes does a candidate need to be a majority candidate?
Round 1

| \# voters | $\mathbf{1 1}$ | $\mathbf{7}$ | $\mathbf{7}$ | $\mathbf{3}$ | $\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st place | Pam | Michelle | Darci | Misty | Michelle |
| 2nd place | Misty | Misty | Michelle | Darci | Darci |
| 3rd place | Michelle | Darci | Misty | Michelle | Misty |
| 4th place | Darci | Pam | Pam | Pam | Pam |

Compute the number of first-place votes for each candidate:
Hint: You already did this on the first page.
Miss Pam:
Miss Misty:
Miss Michelle:
Miss Darci:
Is there a majority candidate in this election in Round 1?
Who is eliminated in Round 1? (Who has the fewest number of first-place votes?)

## Round 2

| \# voters | $\mathbf{1 1}$ | $\mathbf{7}$ | $\mathbf{7}$ | 3 | $\mathbf{1}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1st place | Pam | Michelle | Darci |  |  |
| 2nd place | Michelle | Darci |  |  |  |
| 3rd place | Darci | Pam |  |  |  |

Finish filling in the preference schedule. Then compute the number of first-place votes for each candidate: Miss Pam:

Miss Michelle:
Miss Darci:
Is there a majority candidate in Round 2?
Who is eliminated in Round 2? (Who has the fewest number of first-place votes?)

## Round 3

| \# voters | $\mathbf{1 1}$ | $\mathbf{7}$ | $\mathbf{7}$ | $\mathbf{3}$ | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: |
| 1st place | Pam | Darci | Darci |  |  |
| 2nd place | Darci | Pam |  |  |  |

Finish filling in the preference schedule. Then compute the number of first-place votes for each candidate: Miss Pam:
Miss Darci:
Is there a majority candidate in Round 3?
Who is the winner of Leader of the Year using the Plurality with Elimination Method?

