

## Interest Rate Examples

E.g.1 Suppose a bond mutual fund grows by 10% in 2007, by 7% in 2008, and by 2% in 2009. By how much does the fund grow over the three-year period?

Solution. Let  $P$  denote the value of the fund at the beginning of 2007 and let  $V(t)$  denote the value of the fund  $t$  years after (the beginning of) 2007. So  $V(0) = P$ .

At the end of 2007 (= beginning of 2008), the value of the fund is the original value  $V(0)$  plus 10% of  $V(0)$ , that is:

$$\begin{aligned} V(1) &= V(0) + 0.10 * V(0) \\ &= P + 0.10 P \\ &= P(1 + 0.10) && \text{(factor out } P) \\ &= P(1.10). \end{aligned}$$

At the end of 2008 (= beginning of 2009), the value of the fund is the value at the end of 2007,  $V(1)$ , plus 7% of  $V(1)$ , that is,

$$\begin{aligned} V(2) &= V(1) + 0.07 * V(1) \\ &= V(1) [1 + 0.07] && \text{(factor out } V(1)) \\ &= V(1) (1.07) \\ &= [P(1.10)] (1.07) && \text{(since } V(1) = P(1.10)) \\ &= P(1.10)(1.07). \end{aligned}$$

At the end of 2009 (= beginning of 2010), the value of the fund is the value at the end of 2008,  $V(2)$ , plus 2% of  $V(2)$ , that is,

$$\begin{aligned}V(3) &= V(2) + 0.02 * V(2) \\ &= V(2) [1 + 0.02] \quad (\text{factor out } V(2)) \\ &= V(2) (1.02) \\ &= [P(1.10)(1.07)] (1.02) \quad (\text{since } V(2) = P(1.10)(1.07)) \\ &= P(1.10)(1.07)(1.02) \\ &= 1.20054 P. \quad (\text{multiplying})\end{aligned}$$

The amount of increase in the value of the fund over the three-year period is

$$\begin{aligned}V(3) - V(0) &= 1.20054 P - P \\ &= (1.20054 - 1) P \\ &= 0.20054 P.\end{aligned}$$

therefore, the rate of increase in the value of the fund over the three-year period is

$$\begin{aligned}\frac{\text{amount of increase}}{\text{original amount}} &= \frac{V(3) - V(0)}{V(0)} \\ &= \frac{0.20054 P}{P} \\ &= 0.20054 \\ &= 20.054 \%\end{aligned}$$

Note that this is larger than the sum of the three yearly rates of increase. ■

E.g. 2 A mutual fund grew by 12% in 2000 and by 9% in 2002. How much did it grow in 2001 if it increased by a total of 30% in the three-year period?

Solution. Let  $P$  be the original value of the bond and let  $V(t)$  represent the value  $t$  years after 2000. Let  $r$  be the rate by which the value grows in 2001. Then we have:

$$V(0) = P.$$

$$\begin{aligned} V(1) &= V(0) + 0.12 * V(0) \\ &= V(0) [1 + 0.12] \\ &= V(0) (1.12) \\ &= P (1.12) \end{aligned}$$

$$\begin{aligned} V(2) &= V(1) + r * V(1) \\ &= V(1) [1 + r] \\ &= [P (1.12)] (1 + r) \\ &= P (1.12)(1 + r) \end{aligned}$$

$$\begin{aligned} V(3) &= V(2) + 0.09 * V(2) \\ &= V(2) [1 + 0.09] \\ &= [P (1.12)(1 + r)] (1.09) \\ &= P (1.12)(1 + r)(1.09). \end{aligned}$$

However, we have another way of calculating  $V(3)$ . We are told that the fund grows by a total of 30% over the three-year period. That is,

$$\begin{aligned}
 V(3) &= P + 0.30 * P \\
 &= P(1 + 0.30) \\
 &= P(1.30).
 \end{aligned}$$

We have two expressions for the same quantity,  $V(3)$ , so we will equate them:

$$P(1.12)(1+r)(1.09) = P(1.30).$$

This equation has 2 unknowns,  $P$  and  $r$ , but since  $P \neq 0$ , we may divide both sides by  $P$ , leaving us with one equation in one unknown:

$$\frac{P(1.12)(1+r)(1.09)}{P} = \frac{P(1.30)}{P}$$

$$(1.12)(1+r)(1.09) = 1.30$$

$$\frac{(1.12)(1+r)(1.09)}{(1.12)(1.09)} = \frac{1.30}{(1.12)(1.09)}$$

$$1+r = \frac{1.30}{(1.12)(1.09)}$$

$$1+r - 1 = \frac{1.30}{(1.12)(1.09)} - 1$$

$$r = \frac{1.30}{(1.12)(1.09)} - 1$$

$$r \approx 0.06487549 \dots \approx 0.065$$

So the value of the bond increased by approx. 6.5% in 2001. ■