Geometric Sequences and Sum Formula

Example 1 Consider the geometric sequence with initial term $G_0 = 7$ and common ratio $c = 3$.

(a) Find G_{90}.

Solution Using the explicit formula, we obtain

$$G_{90} = c^{90} P \quad \text{where } c = 3 \text{ and } P = G_0 = 7$$
$$= 3^{90} \times 7 \quad \blacktriangle$$

(b) Give an explicit formula for G_N:

Solution $G_N = 3^N \times 7 \quad \blacktriangle$

(c). Use the Geometric Sum Formula to compute $G_0 + G_1 + \cdots + G_{90}$:

Solution:

$$G_0 + G_1 + \cdots + G_{90} = 7 + 3 \times 7 + \cdots + 3^{90} \times 7$$
$$= 7\left(\frac{3^{91} - 1}{3 - 1}\right)$$
$$= 7 \times \left(\frac{3^{91} - 1}{2}\right) \quad \blacktriangle$$

KEY: (so $N - 1 = 90 \Rightarrow N = 91$)

(d) Use the Geometric Sum Formula to compute $G_{50} + G_{51} + \cdots + G_{90}$.

Solution (This is a different way of looking at it than is given in MyMathLab.)

We note that $G_{50} + G_{51} + \cdots + G_{90}$ is a difference:

$$G_{50} + G_{51} + \cdots + G_{90} = [G_0 + G_1 + \cdots + G_{90}] - [G_0 + G_1 + \cdots + G_{49}].$$

[To see this, write out more of the terms:

$\underbrace{G_0 + G_1 + \cdots + G_{48} + G_{49}}_{\text{subtract this part}} + \underbrace{G_{50} + G_{51} + \cdots + G_{88} + G_{89} + G_{90}}_{\text{to get this part}}$.]

So we have

$$G_{50}+G_{51}+\cdots+G_{90} = [G_0+G_1+\cdots+G_{90}] - [G_0+G_1+\cdots+G_{49}]$$

$$= \left[7 \times \left(\frac{3^{91}-1}{2}\right)\right] - \left[7 \times \left(\frac{3^{50}-1}{2}\right)\right]$$

$$= \left[\frac{7}{2} \times (3^{91}-1)\right] - \left[\frac{7}{2} \times (3^{50}-1)\right]$$

$$= \frac{7}{2}\left[(3^{91}-1)-(3^{50}-1)\right]$$

$$= \frac{7}{2}\left[3^{91}-1-3^{50}+1\right]$$

$$= \frac{7}{2}\left[3^{91}-3^{50}\right]$$

$$= 7 \times \left(\frac{3^{91}-3^{50}}{2}\right).$$ ■

<u>Example 2</u>. Use the Geometric Sum Formula to Compute:

$$\$100(1.075) + \$100(1.075)^2 + \$100(1.075)^3 + \cdots + \$100(1.075)^{23}.$$

<u>Solution</u>. Let us rewrite the sum noting that $P = 100(1.075) = 107.5$. We obtain (suppressing the dollar signs for now):

$$107.5 + 107.5(1.075)^1 + 107.5(1.075)^2 + \cdots + 107.5(1.075)^{22}$$

and so $N-1 = 22$ which means $N = 23$. So the sum equals

$$107.5\left(\frac{1.075^{23}-1}{1.075-1}\right) = 107.5\left(\frac{1.075^{23}-1}{0.075}\right) \quad \left[\begin{array}{l}\text{since}\\ c=1.075\end{array}\right]$$

$$\approx \$6130.50$$

rounded to the nearest penny.

This sort of computation arises in the discussion of Deferred Annuities. ■