Tripling Time

Example. In an investment that has 6.8% APY, how many years will it take to at least triple your money if you invest

(a) \$500

(b) \$P.

(Round up to the next whole year.)

Solution. We know that the future value F of P dollars invested at an APY of R% is $F = P(1+r)^t$ where r denotes R% written as a decimal.

(a) Identify the givens:

$P = 500$

$F = 3 * 500$

$r = 0.068.$

So we wish to solve for t:

$$P(1+r)^t = F$$

$$500(1+0.068)^t = 3 * 500$$

$$1.068^t = 3.$$

Solution 1: using logarithms: We take the natural log of each side:

$$\ln(1.068^t) = \ln 3$$

$$t \cdot \ln(1.068) = \ln 3$$

(Laws of Logs: $\ln A^B = B \ln A$)

$$t = \frac{\ln 3}{\ln 1.068}$$

$$= 16.6993\ldots$$

$$\approx 17 \text{ years.}$$

Solution 2: Trial and Error: If you are unfamiliar with logarithms, use trial and error to find the smallest positive whole number t for which

$$1.068^t \geq 3$$

$1.068^{10} = 1.93068\ldots < 3$

$1.068^{20} = 3.72756\ldots > 3$

$1.068^{15} = 2.6826\ldots < 3$

→ $1.068^{17} = 3.0599\ldots > 3$

$1.068^{16} = 2.865\ldots < 3$

So $t = 17$ years is the solution ▲

(b) Here we are not given a specific numerical value of P; we want $F = 3P$. So we solve for t:

$$P(1 + 0.068)^t = 3P$$

$$\frac{P(1.068)^t}{P} = \frac{3P}{P} \qquad (\text{since } P \neq 0)$$

$$1.068^t = 3.$$

This is the same equation as in part (a), so we see that we triple our money after 17 years if the APY is 6.8%, NO MATTER HOW MUCH WE ORIGINALLY INVESTED. ■