Fibonacci Numbers Exercise Hint

Question Let F_N represent the N^{th} Fibonacci Number. If $F_N = a$ and $F_{N+1} = b$, what is F_{N+M} in terms of a and b?

Hint: We will look for a pattern using the recursive definition $F_N = F_{N-1} + F_{N-2}$.

M=0 : $F_N = a$ } given

M=1 : $F_{N+1} = b$ } given

M=2 :
$$
\begin{aligned}
F_{N+2} &= F_{N+1} + F_N && \text{(recursive rule)} \\
&= b + a && \text{(def}^{\underline{n}}\text{ of a and b)} \\
&= a + b && \text{(rearranging)}
\end{aligned}
$$

M=3 :
$$
\begin{aligned}
F_{N+3} &= F_{N+2} + F_{N+1} && \text{(recursive rule)} \\
&= (a+b) + b && \text{(previous steps)} \\
&= a + 2b && \text{(combining like terms)}
\end{aligned}
$$

M=4 :
$$
\begin{aligned}
F_{N+4} &= F_{N+3} + F_{N+2} && \text{(recursive rule)} \\
&= (a+2b) + (a+b) && \text{(previous steps)} \\
&= 2a + 3b && \text{(combining like terms)}
\end{aligned}
$$

M=5 :
$$
\begin{aligned}
F_{N+5} &= F_{N+4} + F_{N+3} \\
&= (2a+3b) + (a+2b) \\
&= 3a + 5b
\end{aligned}
$$

M=6 :
$$
\begin{aligned}
F_{N+6} &= F_{N+5} + F_{N+4} \\
&= (3a+5b) + (2a+3b) \\
&= 5a + 8b
\end{aligned}
$$

Continue in this way a few more steps, observing the pattern in the coefficients which emerges. Express the general pattern in terms of a, b, and M.