The Golden Property

We define φ (phi) to be the positive solution of the quadratic equation $x^2 = x+1$. Using the quadratic formula, we obtain

$$\varphi = \frac{1+\sqrt{5}}{2}.$$

Now saying that φ is a solution of $x^2 = x+1$ just means that $\varphi^2 = \varphi + 1$. In this way, we are writing the square of φ in terms of φ to the 1st power. Repeated use of this property (the golden property) enables us to write <u>any</u> positive integer power of φ in terms of φ.

φ^2

Golden property: $\varphi^2 = \varphi + 1$

φ^3

mult by φ: $\varphi \cdot \varphi^2 = \varphi(\varphi+1)$

$\varphi^3 = \varphi^2 + \varphi$ but $\varphi^2 = \varphi+1$, so

Golden property $\varphi^3 = (\varphi+1) + \varphi$

$\varphi^3 = 2\varphi + 1$

φ^4

mult by φ: $\varphi \cdot \varphi^3 = \varphi(2\varphi+1)$

$\varphi^4 = 2\varphi^2 + \varphi$ but $\varphi^2 = \varphi+1$, so

golden property: $\varphi^4 = 2(\varphi+1) + \varphi$

$\varphi^4 = 2\varphi + 2 + \varphi$

$\varphi^4 = 3\varphi + 2$

φ^5

mult by φ $\varphi \cdot \varphi^4 = \varphi(3\varphi+2)$

$\varphi^5 = 3\varphi^2 + 2\varphi$ but $\varphi^2 = \varphi+1$, so

$\varphi^5 = 3(\varphi+1) + 2\varphi$

$\varphi^5 = 3\varphi + 3 + 2\varphi$

$\varphi^5 = 5\varphi + 3$

We can continue this way as long as we would like. Do a few more steps and then look at a list of the Fibonacci numbers and try to identify the relationship between them.