

## The Golden Property, II

Recall ① Definition of  $\varphi$  :  $\varphi = \frac{1+\sqrt{5}}{2}$ .

② The Golden Property:  $\varphi^N = F_N \varphi + F_{N-1}$   
where  $F_N$  is the  $N^{\text{th}}$  Fibonacci Number.

Question. Find positive rational numbers  $a$  and  $b$  such that  $\varphi^7 = a\sqrt{5} + b$ .

Solution. From the Golden Property, we have

$$\varphi^7 = F_7 \varphi + F_6 = 13\varphi + 8.$$

Then, by definition of  $\varphi$ ,

$$\varphi^7 = 13 \left( \frac{1+\sqrt{5}}{2} \right) + 8$$

$$= \frac{13}{2} (1+\sqrt{5}) + 8$$

$$= \frac{13}{2} + \frac{13}{2}\sqrt{5} + 8$$

$$= \frac{13}{2}\sqrt{5} + \frac{13}{2} + \frac{16}{2}$$

$$= \frac{13}{2}\sqrt{5} + \frac{29}{2}.$$

So  $a = \frac{13}{2}$  and  $b = \frac{29}{2}$ . ■