Names: $\qquad$ .

## Fair Division Worksheet II

1. Every day, five sisters, Arlene, Babs, Cornelia, Darla, and Eunice, divide fairly 30 pieces of candy, of different types, using the Method of Markers. The candy is placed in an array and the players bid by placing markers as shown below. (Marker $A_{1}$ is Arlene's first marker, $A_{2}$ is her second, $A_{3}$ is her third, and $A_{4}$ is her fourth; the $B_{i}$ are Babs's markers, the $C_{i}$ are Cornelia's; the $D_{i}$ are Darla's; and the $E_{i}$ are Eunice's.) It is agreed that candy left over after the allocation will be saved for the next day. Determine the allocation for each of the following three days. Recall that our convention is to go from left to right. Ties should be broken by alphabetical order.
(a) Which pieces does each person get on Monday?


| player | Arlene | Babs | Cornelia | Darla | Eunice | Leftovers |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| pieces received |  |  |  |  |  |  |

(b) Which pieces does each person get on Tuesday?

(c) Which pieces does each person get on Wednesday?

2. The sisters, Arlene, Babs, Cornelia, Darla, and Eunice, have inherited a faux mink stole and a cubic zirconium tiara from their Great Aunt Fiona. They decide to divide the estate using the Method of Sealed Bids. The players' bids are given in the table below.
(a) (The Bids) Determine each player's opinion of the total value of the estate and the value that each player places on a fair share. Enter these values in rows (1) and (2) of the table.
(b) (The Allocation) Determine the allocation of the items in the estate. Fill in the item(s) allocated to each player in row (3) of the table.
(c) (The Payments) After the items are allocated, some players will owe the estate money and others will be owed money by the estate. Determine the amount of money each player owes or is owed and enter this in row (4) $O R$ (5) of the table.
(d) (Dividing the Surplus) After the payments are all made, there might be a surplus left in the estate. Determine each player's share of the surplus and enter it in row (6) of the table.
(e) (Final Settlement) Find the net settlement (items and money) for each player and enter it in row (7) of the table.

|  |  |  | Arlene | Babs | Cornelia | Darla | Eunice |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Bids |  | Stole | \$80 | \$44 | \$32 | \$60 | \$100 |
|  |  | Tiara | \$50 | \$36 | \$28 | \$45 | \$40 |
|  | (1) | Total Value |  |  |  |  |  |
|  | (2) | Fair Share |  |  |  |  |  |
| Allocation | (3) | Item(s) |  |  |  |  |  |
| Payments | (4) | Player Owes Estate |  |  |  |  |  |
|  | (5) | Estate Owes Player |  |  |  |  |  |
| Surplus | (6) | Share of Surplus |  |  |  |  |  |
| Final Settlement | (7) |  |  |  |  |  |  |

3. Every day for lunch, three brothers, Xerxes, Yuri, and Zebulon, buy a cheese foot-long submarine sandwich for $\$ 6$ to share. It is half Stilton and half Gouda. Xerxes likes Stilton and Gouda equally well. Yuri likes Stilton twice as much as Gouda. Zebulon likes Gouda twice as much as Stilton. They plan to divide the sandwich using the Lone Divider Method. The divider must make crosswise cuts, as few as possible.
(a) On Monday, Xerxes is the divider. Explain where Xerxes would cut the sandwich. (Draw a picture.)
(b) Determine the value of each of the pieces Xerxes has cut to Yuri and to Zebulon. Give the bid list for each player.
(c) Is it possible to distribute the pieces cut by Xerxes so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Yuri as the second divider. In either case, describe the piece each player gets in the end (how many inches of each type of cheese and the dollar amount).
(d) On Tuesday, Zebulon is the divider. Explain where Zebulon would cut the sandwich. (Draw a picture.)
(e) Determine the value of each of the pieces Zebulon has cut to Xerxes and to Yuri. Give the bid list for each player.
(f) Is it possible to distribute the pieces cut by Zebulon so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Xerxes as the second divider. In either case, describe the piece each player gets in the end (how many inches of each type of cheese and the dollar amount).
4. Every day for dinner, the three brothers, Xerxes, Yuri, and Zebulon, buy a round pizza for $\$ 21$ to share. It is one-third sausage, one-third mushroom, and one-third anchovy. Xerxes likes all three toppings equally well. Yuri likes mushroom twice as much as sausage and anchovies twice as much as mushrooms. Zebulon likes sausage and mushroom equally well and anchovies five times as much as sausage. They plan to divide the pizza using the Lone Divider Method. The divider must make cuts from the middle (as one usually does with a round pizza), as few as possible.
(a) On Monday, Xerxes is the divider. Explain all the different ways that Xerxes might cut the pizza. (Draw pictures.)
(b) Suppose Xerxes cuts the pizza along the boundaries between the toppings. (So one piece is all sausage, the second is all mushroom, and the third is all anchovy.) Determine the value of each of the pieces Xerxes has cut to Yuri and to Zebulon. Give the bid list for each player.
(c) Is it possible to distribute the pieces cut by Xerxes so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Yuri as the second divider. In either case, describe the piece each player gets in the end (how many degrees of each topping and the dollar amount).
(d) On Tuesday, Zebulon is the divider. Find one way (any way) that Zebulon might cut the pizza. (Draw a picture.)
(e) Determine the value of each of the pieces Zebulon has cut to Xerxes and to Yuri. Give the bid list for each player.
(f) Is it possible to distribute the pieces cut by Zebulon so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Xerxes as the second divider. In either case, describe the piece each player gets in the end (how many degrees of each topping and the dollar amount).
