

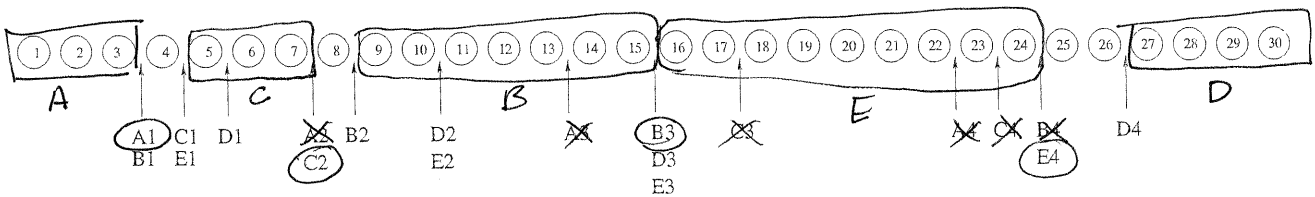
Names: _____

KEY

Fair Division Worksheet II

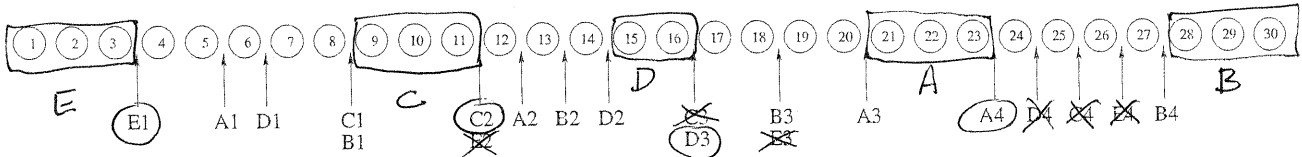
1. Every day, five sisters, Arlene, Babs, Cornelia, Darla, and Eunice, divide fairly 30 pieces of candy, of different types, using the **Method of Markers**. The candy is placed in an array and the players bid by placing markers as shown below. (Marker A_1 is Arlene's first marker, A_2 is her second, A_3 is her third, and A_4 is her fourth; the B_i are Babs's markers, the C_i are Cornelia's; the D_i are Darla's; and the E_i are Eunice's.) It is agreed that candy left over after the allocation will be saved for the next day. Determine the allocation for each of the following three days. Recall that our convention is to go from left to right. Ties should be broken by alphabetical order.

(a) Which pieces does each person get on Monday?



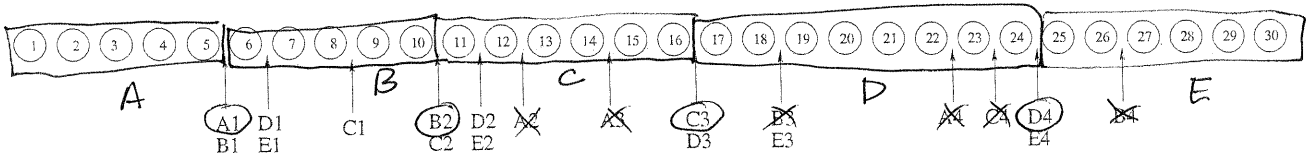
player	Arlene	Babs	Cornelia	Darla	Eunice	Leftovers
pieces received	1-3	9-15	5-7	27-30	16-24	4, 8, 25, 26

(b) Which pieces does each person get on Tuesday?



player	Arlene	Babs	Cornelia	Darla	Eunice	Leftovers
pieces received	21-23	28-30	9-11	15-16	1-3	4-8; 12-14; 17-20; 24-27

(c) Which pieces does each person get on Wednesday?



player	Arlene	Babs	Cornelia	Darla	Eunice	Leftovers
pieces received	1-5	6-10	11-16	17-24	25-30	X

2. The sisters, Arlene, Babs, Cornelia, Darla, and Eunice, have inherited a faux mink stole and a cubic zirconium tiara from their Great Aunt Fiona. They decide to divide the estate using the **Method of Sealed Bids**. The players' bids are given in the table below.

(a) **(The Bids)** Determine each player's opinion of the total value of the estate and the value that each player places on a fair share. Enter these values in rows (1) and (2) of the table.

(b) **(The Allocation)** Determine the allocation of the items in the estate. Fill in the item(s) allocated to each player in row (3) of the table.

(c) **(The Payments)** After the items are allocated, some players will owe the estate money and others will be owed money by the estate. Determine the amount of money each player owes or is owed and enter this in row (4) OR (5) of the table.

(d) **(Dividing the Surplus)** After the payments are all made, there might be a surplus left in the estate. Determine each player's share of the surplus and enter it in row (6) of the table.

$$\frac{\text{surplus}}{5} = \frac{24 + 72 - 16 - 12 - 21}{5} = \frac{47}{5} = \$9.40$$

(e) **(Final Settlement)** Find the net settlement (items and money) for each player and enter it in row (7) of the table.

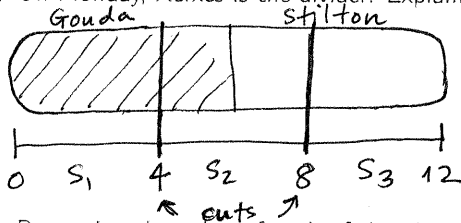
		Arlene	Babs	Cornelia	Darla	Eunice
Bids	Stole	\$80	\$44	\$32	\$60	\$100
	Tiara	\$50	\$36	\$28	\$45	\$40
	(1) Total Value	\$130	\$80	\$60	\$105	\$140
	(2) Fair Share	\$26	\$16	\$12	\$21	\$28
Allocation	(3) Value of Item(s)	\$50	\$0	\$0	\$0	\$100
Payments	(4) Player Owes Estate	\$24	X	X	X	\$72
	(5) Estate Owes Player	X	\$16	\$12	\$21	X
Surplus	(6) Share of Surplus	\$9.40	\$9.40	\$9.40	\$9.40	\$9.40
Final Settlement	(7)	Tiara pays \$14.60	\$25.40	\$21.40	\$30.40	Stole pays \$62.60

Fair share is worth

$$\frac{\$6}{3} = \$2$$

3. Every day for lunch, three brothers, Xerxes, Yuri, and Zebulon, buy a cheese foot-long submarine sandwich for \$6 to share. It is half Stilton and half Gouda. Xerxes likes Stilton and Gouda equally well. Yuri likes Stilton twice as much as Gouda. Zebulon likes Gouda twice as much as Stilton. They plan to divide the sandwich using the Lone Divider Method. The divider must make crosswise cuts, as few as possible.

- (a) On Monday, Xerxes is the divider. Explain where Xerxes would cut the sandwich. (Draw a picture.)



Bid Lists:
 Xerxes: S_1, S_2, S_3
 Yuri: S_2, S_3
 Zebulon: S_1, S_2

Z
 $S_1: 4(2z) = 8z = \frac{8}{3}$
 $S_2: 2(2z) + 2z = 6z = \frac{6}{3} = 2$
 $S_3: 4z = \frac{4}{3}$

- (b) Determine the value of each of the pieces Xerxes has cut to Yuri and to Zebulon. Give the bid list for each above.

Yuri: $y: 1 \text{ m gouda}$
 $2y: 1 \text{ m stilton}$
 $6(y) + 6(2y) = 6.00$
 $6y = 6.00$
 gouda: $y = \frac{1}{3}$
 Stilton: $2y = \frac{2}{3}$

Y
 $S_1: 4y = 4(\frac{1}{3}) = \frac{4}{3}$
 $S_2: 2y + 2(2y) = 6y = \frac{6}{3} = 2$
 $S_3: 4(2y) = 8y = \frac{8}{3}$

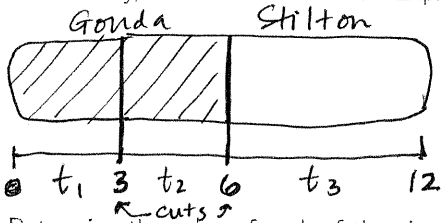
Zebulon
 $Z: 1 \text{ m Stilton}$
 $2Z: 1 \text{ m Gouda}$
 $6(2z) + 6(z) = 6.00$
 $18z = 6.00$
 Stilton $Z = \frac{1}{3}$
 gouda $2z = \frac{2}{3}$

- (c) Is it possible to distribute the pieces cut by Xerxes so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Yuri as the second divider. In either case, describe the piece each player gets in the end (how many inches of each type of cheese and the dollar amount).

Yes, there are 3 ways to do this:

	①	②	③
X	S_3	S_1	S_2
Y	S_2	S_3	S_3
Z	S_1	S_2	S_1

- (d) On Tuesday, Zebulon is the divider. Explain where Zebulon would cut the sandwich. (Draw a picture.)



Bid Lists:
 X: t_3
 Y: t_3
 Z: t_1, t_2, t_3

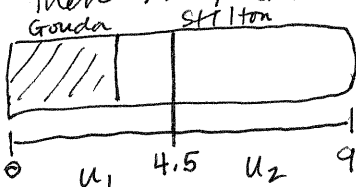
- (e) Determine the value of each of the pieces Zebulon has cut to Xerxes and to Yuri. Give the bid list for each above.

Xerxes
 $t_1: 3(\frac{1}{2}) = \frac{3}{2} = \1.50
 $t_2: \$1.50$
 $t_3: 6(\frac{1}{2}) = \$3.00$

Yuri
 $t_1: 3(\frac{1}{3}) = \$1$
 $t_2: 3(\frac{1}{3}) = \$1$
 $t_3: 6(\frac{2}{3}) = \$4$

- (f) Is it possible to distribute the pieces cut by Zebulon so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Xerxes as the second divider. In either case, describe the piece each player gets in the end (how many inches of each type of cheese and the dollar amount).

NO fair distribution of t_1, t_2, t_3 . Give t_1 (containing t_2) to Zebulon. Then X & Y divide what's left. Xerxes cuts into 2 pieces, each 4.5 in, worth $\frac{1}{2}(4.50) = \$2.25$ each to Xerxes.



Yuri:
 $u_1: 3(\frac{1}{3}) + 1.5(\frac{2}{3}) = 1 + 1 = \2
 $u_2: 4.5(\frac{2}{3}) = \frac{9}{2} \cdot \frac{2}{3} = \3

So Yuri picks u_2 and Xerxes gets u_1 .

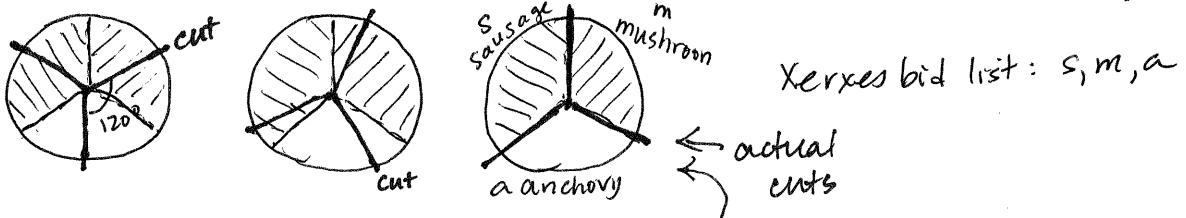
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Fair Share:

$$\frac{\$21}{3} = \$7$$

4. Every day for dinner, the three brothers, Xerxes, Yuri, and Zebulon, buy a round pizza for $\$21$ to share. It is one-third sausage, one-third mushroom, and one-third anchovy. Xerxes likes all three toppings equally well. Yuri likes mushroom twice as much as sausage and anchovies twice as much as mushrooms. Zebulon likes sausage and mushroom equally well and anchovies five times as much as sausage. They plan to divide the pizza using the **Lone Divider Method**. The divider must make cuts from the middle (as one usually does with a round pizza), as few as possible.

(a) On Monday, Xerxes is the divider. Explain all the different ways that Xerxes might cut the pizza. (Draw pictures.) Xerxes cuts it anywhere first, then makes 2 more cuts 120° away.



(b) Suppose Xerxes cuts the pizza along the boundaries between the toppings. (So one piece is all sausage, the second is all mushroom, and the third is all anchovy.) Determine the value of each of the pieces Xerxes has cut to Yuri and to Zebulon. Give the bid list for each player.

Yuri: Let y be value of sausage part;
 then $2y$ is value of mushroom;
 $2(2y) = 4y$ value of anchovies;
 So $y + 2y + 4y = 21$
 $7y = 21$
 $y = \$3$
 $m \quad 2y = \$6$
 $a \quad 4y = \$12$

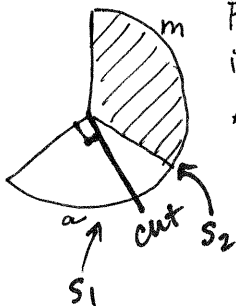
Yuri's bid list: a

Zebulon: let z be value of sausage part;
 z value of mushroom
 $5z$ value of anchovy
 So $z + z + 5z = 21$
 $7z = 21$
 $z = \$3$
 $s, m \quad z = \$3$
 $a \quad 5z = \$15$

Zebulon's bid list: a

(c) Is it possible to distribute the pieces cut by Xerxes so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Yuri as the second divider. In either case, describe the piece each player gets in the end (how many degrees of each topping and the dollar amount).

No. We could give Xerxes s or m . Let's give him s . Then Y & Z divide them and a parts left.



For Y , this is worth $\$6 + \$12 = \$18$; so he must divide it into 2 pieces worth $\$9$ each.

Anchovies: $\frac{\$9}{\$12} = \frac{3}{4}$ so $\frac{3}{4}(120^\circ) = 90^\circ$.

Zebulon: $s_1: \left(\frac{90^\circ}{120^\circ} = \frac{3}{4}\right)$ so $\frac{3}{4}(15) = \frac{45}{4} = \11.25

$s_2: \left(\frac{30^\circ}{120^\circ} = \frac{1}{4}\right)$ so $\frac{1}{4}(15) + 3 = 3.75 + 3 = \6.75

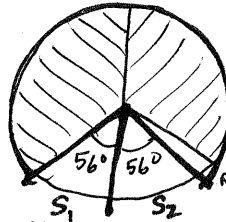
continued on next page \uparrow anchovies \uparrow mushrooms

So Z takes s_1 and Y gets s_2 .

(d) On Tuesday, Zebulon is the divider. Find one way (any way) that Zebulon might cut the pizza. (Draw a picture.) How much anchovy is worth \$7 to Z?

$$\frac{\$7}{\$15} = \frac{7}{15} \text{ so } \frac{7}{15} (120^\circ) = 56^\circ$$

Zebulon bid list: S_1, S_2, S_3



80° of anchovy + all rest

(e) Determine the value of each of the pieces Zebulon has cut to Xerxes and to Yuri. Give the bid list for each player.

Xerxes $S_1: \frac{56^\circ}{120^\circ} = \frac{7}{15}$

so $\frac{7}{15} (\$7) = \frac{\$49}{15}$

$S_2: \frac{\$49}{15} \approx \3.27

$S_3: \$7 + \$7 + \frac{\$7}{15} = \frac{\$217}{15} \approx \$14.47$

Yuri $S_1: \frac{7}{15} (\$12) = \5.60

$S_2: \$5.60$

$S_3: \frac{1}{15} (\$12) + \$3 + \$6 = \9.80

Bid List: S_3

anchovy sliver:

$(\frac{8^\circ}{120^\circ}) = \frac{1}{15}$

$\frac{1}{15} (\$7) = \frac{\$7}{15}$

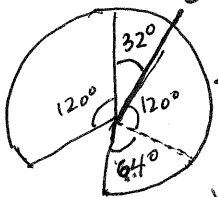
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Bid List: S_3

(f) Is it possible to distribute the pieces cut by Zebulon so that the result is a fair division? If so, how? Otherwise, explain how to finish the division using Xerxes as the second divider. In either case, describe the piece each player gets in the end (how many degrees of each topping and the dollar amount).

NO. Give Z S_1 or S_2 (say S_1). Then let X divide remainder into two equal (to him) pieces. Rest is worth $\$17.73$,

t_1 cut so we want each to be worth $\$8.86$. He values it all equally, so cut into two pieces of equal size.



t_2 Total: $\frac{360^\circ - 56^\circ}{2} = \frac{304^\circ}{2} = \underline{\underline{152^\circ}}$

Yuri $t_1: \$3 + (\frac{32^\circ}{120^\circ}) (\$6) = \$3 + \$1.60 = \$4.60$

$t_2: (\frac{88^\circ}{120^\circ}) (\$6) + (\frac{64^\circ}{120^\circ}) (\$12) = \$4.40 + \$6.40 = \$10.80$

So Yuri takes t_2 and Xerxes gets t_1 .