

Names: KEY

Fair Division Worksheet

1. A round pizza is half cheese and half pepperoni. It costs \$7.20. Walter values pepperoni four times as much as cheese. Jesse values cheese twice as much as pepperoni.

(a) Find the value of the cheese half of the pizza, according to Walter's value system. Round to the nearest cent.

Let  $x$  be value of cheese half to Walter.

Then  $4x + x = 7.20$   
 $5x = 7.20$   
 $x = 1.44$       \$ 1.44

(b) Find the value of the pepperoni half of the pizza, according to Walter's value system. Round to the nearest cent.

$4x = 4(1.44) = \underline{\underline{\$ 5.76}}$

(c) Find the value of a 45° cheese slice of pizza, according to Walter's value system. Round to the nearest cent.

$\frac{45^\circ}{180^\circ} = \frac{1}{4}$       So       $\frac{1}{4}(1.44) = \underline{\underline{\$ 0.36}}$

(d) Find the value of a 60° pepperoni slice of pizza, according to Walter's value system. Round to the nearest cent.

$\frac{60^\circ}{180^\circ} = \frac{1}{3}$       So       $\frac{1}{3}(5.76) = \underline{\underline{\$ 1.92}}$

(e) Find the value of the cheese half of the pizza, according to Jesse's value system. Round to the nearest cent.

Let  $y$  be value of pepperoni half to Jesse.

Then  $2y + y = 7.20$   
 $3y = 7.20$   
 $y = 2.40$        $2(2.40) = \underline{\underline{\$ 4.80}}$

(f) Find the value of the pepperoni half of the pizza, according to Jesse's value system. Round to the nearest cent.

\$ 2.40

(g) Find the value of a 45° cheese slice of pizza, according to Jesse's value system. Round to the nearest cent.

$\frac{45^\circ}{180^\circ} = \frac{1}{4}$       So       $\frac{1}{4}(4.80) = \underline{\underline{\$ 1.20}}$

(h) Find the value of a 60° pepperoni slice of pizza, according to Jesse's value system. Round to the nearest cent.

$\frac{60^\circ}{180^\circ} = \frac{1}{3}$       So       $\frac{1}{3}(2.40) = \underline{\underline{\$ 0.80}}$

2. Six friends agree to divide a \$18 pizza fairly using the **Lone Divider Method**. The table shows how each player values each of the six slices that have been cut by the divider. Assume that all of the friends play honestly.

	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$	$s_6$
Tuco	\$1	\$2	\$1	\$1	\$1	\$13
Hector	\$3	\$3	\$3	\$3	\$3	\$3
Leonel	\$8	\$2	\$2	\$2	\$2	\$2
Marco	\$5	\$2	\$3	\$2	\$2	\$4
Tortuga	\$2	\$2	\$8	\$1	\$3	\$2
Don Eliado	\$10	\$1	\$0	\$4	\$0	\$3

- (a) How much is a fair share worth?  $\frac{\$18}{6} = \$3$
- (b) Which player was the Divider? *Hector*
- (c) Give the bid list (declaration) for each player.

player	Tuco	Hector	Leonel	Marco	Tortuga	Don Eliado
bid list	$s_6$	all	$s_1$	$s_1, s_3, s_6$	$s_3, s_5$	$s_1, s_4, s_6$

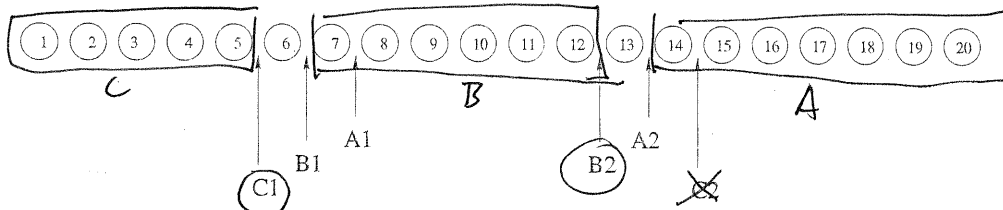
- (d) Give a possible Fair Division of the pizza, if one exists.

player	Tuco	Hector	Leonel	Marco	Tortuga	Don Eliado
slice received	$s_6$	$s_2$	$s_1$	$s_3$	$s_5$	$s_4$

- (e) What would the players do if there were no possible fair division of these slices?

*Give a piece nobody else wants to Hector; put the rest back together and start again with a 5-person Lone-Divider Game.*

3. Three students, Andrea, Brock, and Combo, divide fairly 20 pieces of candy, of different types, using the **Method of Markers**. The candy is placed in an array and the players bid by placing markers as shown below. (Marker  $A_1$  is Andrea's first marker,  $A_2$  is her second;  $B_1$  and  $B_2$  are Brock's markers, and  $C_1$  and  $C_2$  are Combo's.) It is agreed that candy leftover after the allocation will be put aside for a rainy day *Recall that our convention is to go from left to right.*



Which pieces does each person get?

player	Andrea	Brock	Combo	Leftovers
pieces received	14-20	7-12	1-5	6, 13

4. Four heirs, Skyler, Flynn, Holly, and Marie divide fairly an estate consisting of four items— a car wash, a house, a car, and a watch— using the **Method of Sealed Bids**. The players' bids are given in the table below.

(a) **(The Bids)** Determine each player's opinion of the total value of the estate and the value that each player places on a fair share. Enter these values in rows (1) and (2) of the table.

(b) **(The Allocation)** Determine the allocation of the items in the estate. Fill in the item(s) allocated to each player in row (3) of the table.

(c) **(The Payments)** After the items are allocated, some players will owe the estate money and others will be owed money by the estate. Determine the amount of money each player owes or is owed and enter this in row (4) OR (5) of the table.

(d) **(Dividing the Surplus)** After the payments are all made, there might be a surplus left in the estate. Determine each player's share of the surplus and enter it in row (6) of the table.

$$\frac{\text{Surplus}}{4} = \frac{557,250 - 156,125 - 1,800 - 71,525}{4} = \frac{327,800}{4} = \$81,950$$

(e) **(Final Settlement)** Find the net settlement (items and money) for each player and enter it in row (7) of the table.

		Skyler	Flynn	Holly	Marie
Bids	Car Wash	\$800,000	\$500,000	\$750,000	\$100,000
	House	\$200,000	\$120,000	\$250,000	\$180,000
	Car	\$10,000	\$9,000	\$6,000	\$5,000
	Watch	\$1,000	\$1,500	\$1,200	\$1,100
	(1) Total Value of Estate	1,041,000	630,500	1,007,200	286,100
	(2) Value of a Fair Share	252,750	157,625	251,800	71,525
Allocation	(3) Value of Item(s) Allocated	810,000	1,500	250,000	0
Payments	(4) Player Owes Estate	557,250	X	X	X
	(5) Estate Owes Player	X	156,125	1,800	71,525
Surplus	(6) Share of Surplus	81,950	81,950	81,950	81,950
Final Settlement	(7) Final Settlement	car wash car <u>PAYS</u> \$475,300	watch \$238,075	house \$83,750	\$153,475

→ Fair Share is worth \$2.40.

5. Saul and Huell buy a 12-inch half-ham, half-turkey sub sandwich for \$4.80. (So there are 6 inches of ham and 6 inches of turkey.) Saul values turkey three times as much as ham. Huell values ham five times as much as turkey.

(a) Find the value of 1 inch of ham and 1 inch of turkey to Saul.

Let  $x$  be value of 1 in ham to Saul; so turkey is worth  $\$3x$  per in.

Then  $6x + 6(3x) = 4.80$   
 $6x + 18x = 4.80$   
 $24x = 4.80$

turkey:  $3x = \underline{\underline{\$0.60 \text{ per in}}}$

$x = \underline{\underline{\$0.20 \text{ (ham)}}}$

(b) Find the value of 1 inch of ham and 1 inch of turkey to Huell.

Let  $y$  be value of 1 in turkey to Huell; so ham is worth  $5y$  per in.

Then  $6y + 6(5y) = 4.80$   
 $6y + 30y = 4.80$   
 $36y = 4.80$

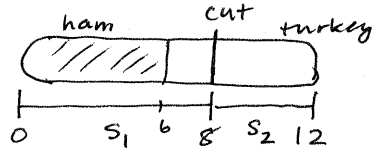
ham:  $5y = \underline{\underline{\$0.6\bar{6} \text{ per in}}}$

$y = \underline{\underline{\$0.1\bar{3} \text{ (turkey)}}}$

(c) If Saul were the divider in a two-person **Divider-Chooser** game, where would he cut the sandwich? (Suppose he must make just one cut, cross-wise.)

How many inches of turkey is worth \$2.40?

$\frac{2.40}{0.60} = 4 \text{ inches.}$



(d) How much would each of the pieces Saul cut be worth to Huell?

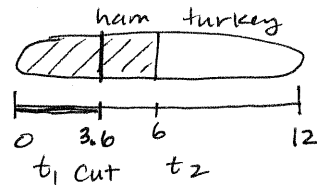
To Huell,  $s_2$  is worth  $4(0.1\bar{3}) = \underline{\underline{\$0.5\bar{3}}}$

So  $s_1$  is worth  $4.80 - 0.5\bar{3} = \underline{\underline{\$4.2\bar{6}}}$

(e) If Huell were the divider in a two-person **Divider-Chooser** game, where would he cut the sandwich? (Suppose he must make just one cut, cross-wise.)

How many inches of ham is worth \$2.40 to Huell?

$\frac{2.40}{0.6\bar{6}} = \frac{2.40}{2/3} = \frac{2.40}{1} \cdot \frac{3}{2} = 3.6 \text{ inches}$



(f) How much would each of the pieces Huell cut be worth to Saul?

To Saul,  $t_1$  is worth  $3.6(0.20) = \underline{\underline{\$0.72}}$

So  $t_2$  is worth  $4.80 - \$0.72 = \underline{\underline{\$4.08}}$