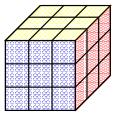
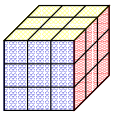
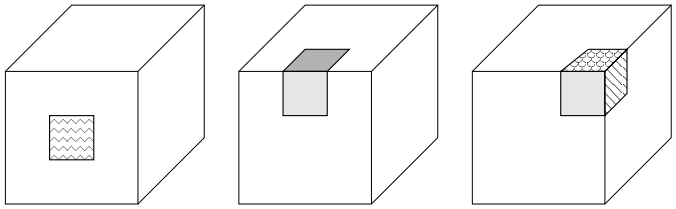


Solving Rubik's Cube



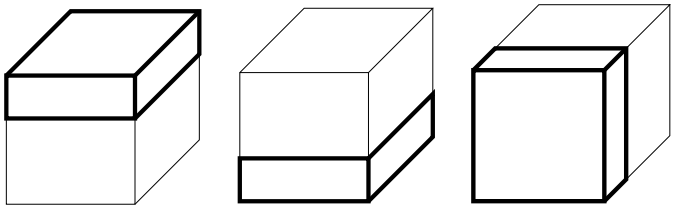
Based on notes by Morley Davidson
Adapted and illustrated by Darci L. Kracht
Kent State University
October 29, 2012

We will start by introducing some convenient terminology. There are three types of cubies: Center Cubies, Edge Cubies, and Corner Cubies. (How many of each are there?)



Center Cubie Edge Cubie Corner Cubie

It will be convenient to name some of the layers of the cube. You can orient your cube any way you like. I suggest always choosing the same color for the Upper Layer (top) so that you don't get mixed up. Many people choose the color which has the logo for the Upper Layer (white on the standard Rubik's Cube).



Upper Layer Down Layer Front Layer

Solving the Upper Layer

The approach that we will take is to solve the Upper Layer first. We will do this in two steps: Solving the Upper Edges and Solving the Upper Corners. The algorithms provided here keep everything else in the Upper Layer fixed, so that you can solve all Upper Edges first, then all Upper Corners; or you can solve all Upper Corners first, followed by all Upper Edges; or you can solve some Upper Edges, then some Corners, then some Edges, then some Corners, etc.

The algorithms that we will give for solving the Upper Layer will be used to build the algorithms for solving the Middle and Last Layers as well. It is hoped that with experience, you will come to understand why these algorithms work.

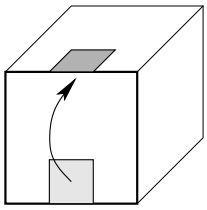
Step 0: Getting Ready

There are far too many possible cases to include here. If the cubie you want to move is not in one of the positions shown, you may have to rotate the whole cube in space (keeping the Upper Layer on top) and/or turn the Upper or Down Layer to get the cubie into position. If you turn the Upper Layer, you will have to turn it the opposite way at the end to line up the centers of the Front, Back, Left, and Right Layers with the corresponding colors on the Upper Layer.

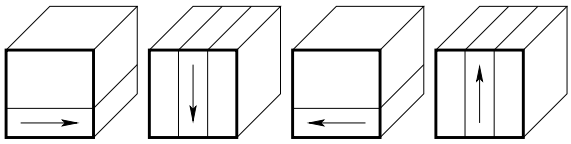
Please note that in the diagrams that follow, all moves are viewed from the **FRONT**.

Step 1: Upper Edges

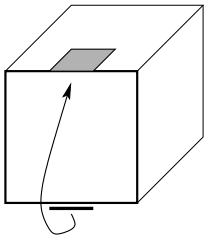
Case I.



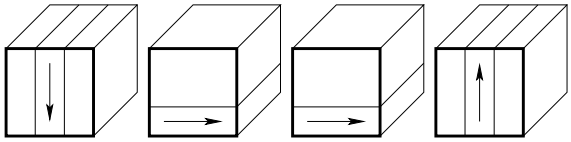
Moves:



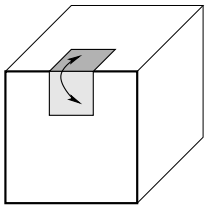
Case II.



Moves:

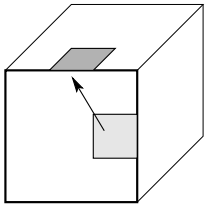


Case III.

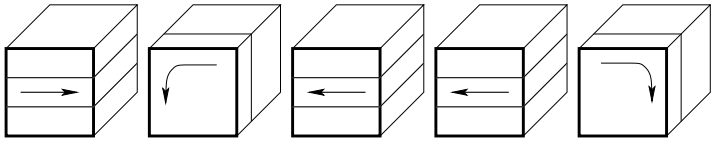


Do a **Case II** followed by a **Case I**.

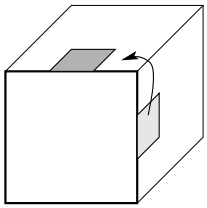
Case IV.



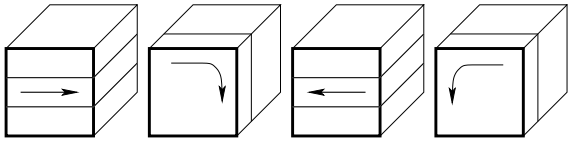
Moves:



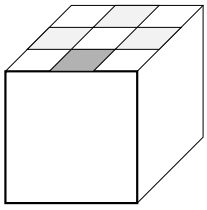
Case V.



Moves:

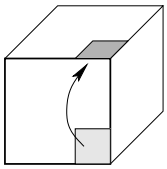
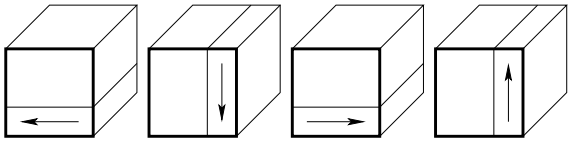


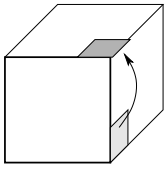
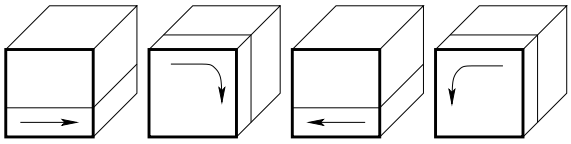
Case VI.

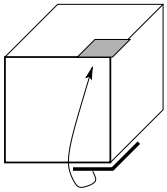
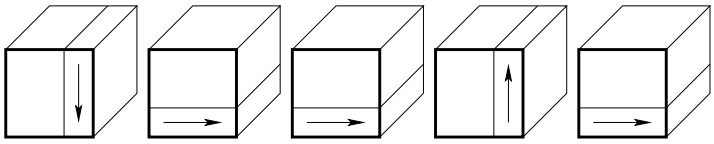


If the desired edge cubie is already on the Upper Layer, but not on the Front Layer, then rotate the whole cube so that it is on the Front Layer. Then use **Case I** to move it (by replacing it) to the Down Layer. Then use **Case I** or **Case II** to correctly place it.

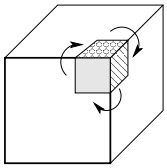
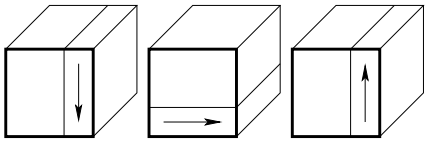
Step 2: Upper Corners

Case I.  Moves: 

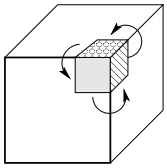
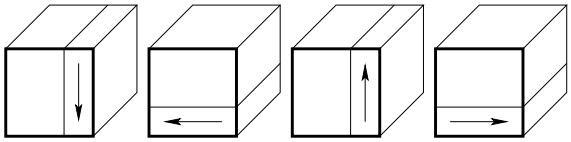
Case II.  Moves: 

Case III.  Moves: 

followed by **Case II**.

Case IV.  Moves: 

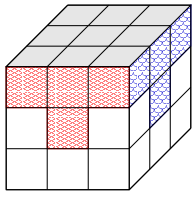
followed by **Case I**.

Case V.  Moves: 

followed by **Case II**.

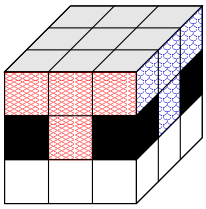
Case VI. Analogous to the last Case of Step 1: Use a **Case I** to move the desired cubie to the Down Layer, and then use the appropriate Case to correctly place it.

After solving the Upper Layer, the top face of your cube should be all one color and each of the four side faces should have a "T" shape of one color as follows (with a different color on each of the four sides, of course).



Solving the Middle Layer

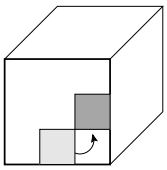
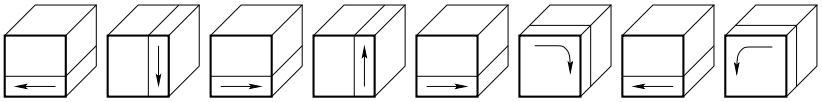
Next we wish to solve the Middle Layer. Since each of the four Center Cubies is already in place, it suffices to solve the four Edge Cubies. Three of them are visible here (in black):

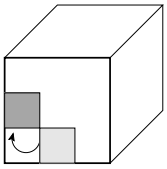
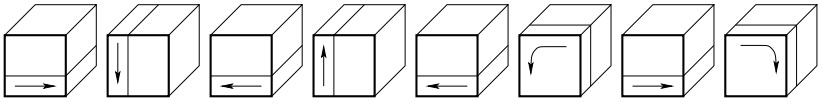


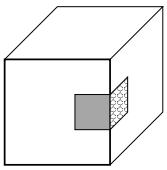
The algorithms that we will give for solving the Middle Layer will preserve the Upper Layer as well as anything else in the Middle Layer that we have already solved.

Step 3: Middle Edges

In the diagrams that follow, all moves are again viewed from the **FRONT**. Note that Case II is the mirror image right-to-left of Case I. Remember that preliminary rotations of the whole cube in space and/or turns of the Down Layer might be needed to put yourself into one the cases below.

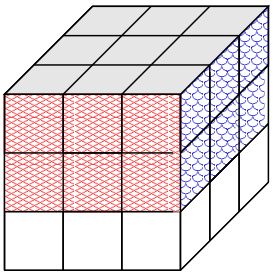
Case I.  Moves: 

Case II.  Moves: 

Case III. 

If the desired edge cubie is already on the Middle Layer, but either positioned incorrectly or oriented incorrectly ("flipped"), then use Case I or Case II (it doesn't matter which) to knock it down to the Down Layer, and then use Case I or Case II *as appropriate*. (Time can be saved by correctly solving the Middle Edge position affected in the initial "bogus" application of Case I/II.)

After solving the first and second layers, your cube should look something like this:

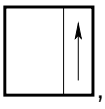
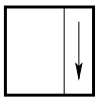


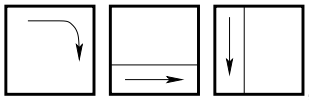
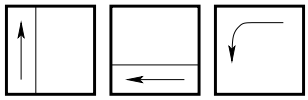
Solving the Last Layer

To solve the Last Layer, we introduce the mathematical idea of *inverses*.

Inverses

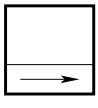
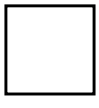
If X is a move or a sequence of moves, then the **inverse** of X, denoted by X', is the move or sequence of moves that "undoes" X. For example:

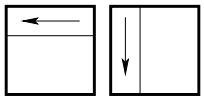
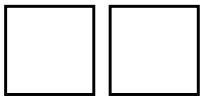
1. If $R =$  , then $R' =$  .

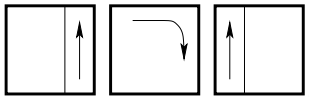
2. If $Y =$  , then $Y' =$  .

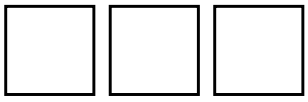
Note that in the second example, the moves are the turns in the opposite direction *in the opposite order*. To see why this is so, consider putting on your shoes and socks. In the morning you put on your socks first, then your shoes. But in the evening, you must take off your shoes first, then your socks.

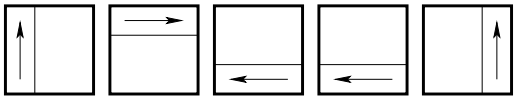
Exercise. Fill in the inverses for each of the following. MOVES ARE STILL VIEWED FROM THE **FRONT**.

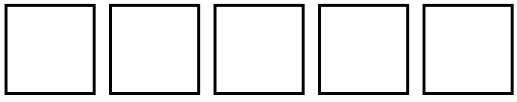
1. If $D =$  , then $D' =$  .

2. If $A =$  , then $A' =$  .

3. If $B =$  ,

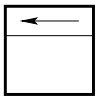
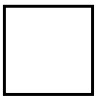
   then $B' =$  .

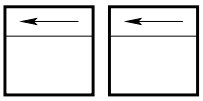
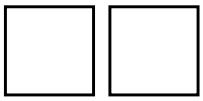
4. If $C =$  ,

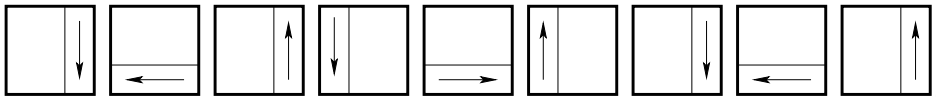
   then $C' =$  .

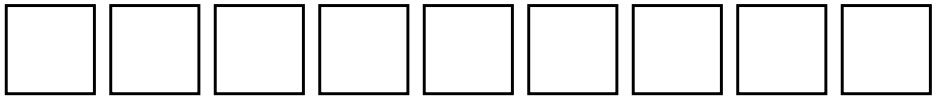
Sequences of Moves Needed to Solve the Last Layer

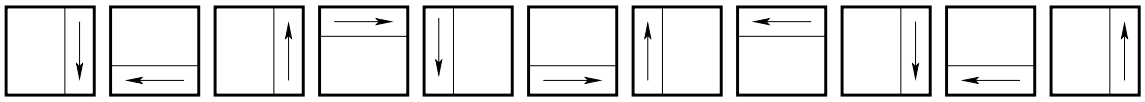
Fill in the inverses for each of the following. MOVES ARE STILL VIEWED FROM THE **FRONT**.

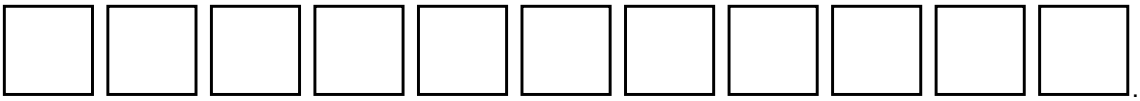
1. If $U =$  , then $U' =$  .

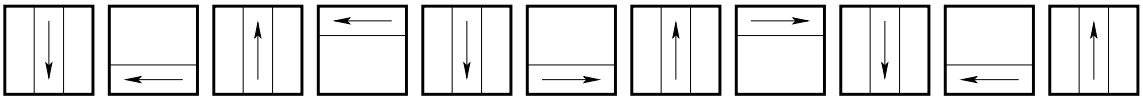
2. If $U^2 =$  , then $(U^2)' =$  .

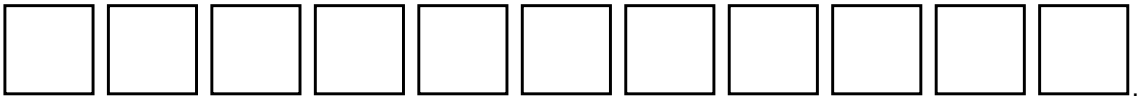
3. If $S_c =$  ,

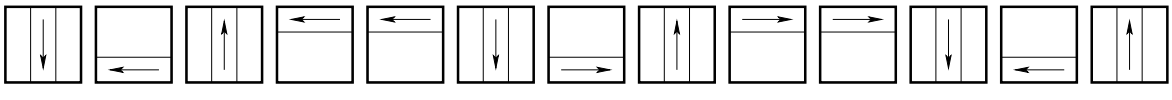
   then $S_c' =$  .

4. If $\widehat{S}_c =$  ,

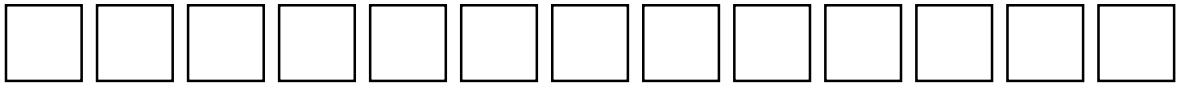
   then $\widehat{S}_c' =$  .

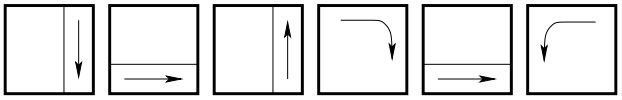
5. If $S_e =$  ,

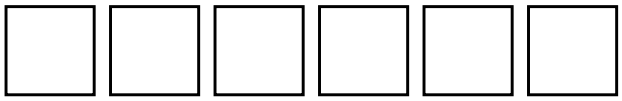
   then $S_e' =$  .

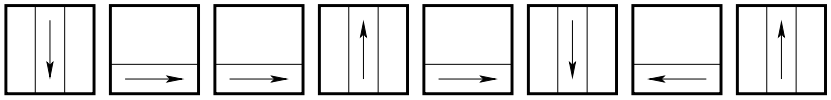
6. If $\widehat{S}_e =$  ,

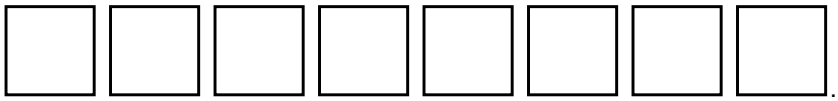
   then

   $\widehat{S}_e' =$  .

7. If $T_c =$  ,

then $T_c' =$  .

8. If $T_e =$  .

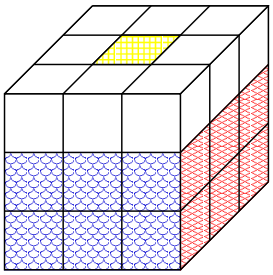
then $T_e' =$  .

(MOVES ABOVE ARE STILL VIEWED FROM THE **FRONT**.)

Now, we return to solving the Last Layer. At this point, all of the cubies that belong in the Last Layer are in the Last Layer. However, most of them are probably in the wrong position and/or have the wrong orientation. Our challenge now is to reposition and reorient them without changing the two layers that we've already solved. The four steps that follow can be done in any order. But first,

TURN THE CUBE OVER!

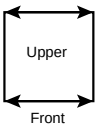
Now the layer to be solved has become the Upper Layer as follows:



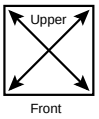
The letters in the steps that follow denote moves or sequences of moves given in the list above. The moves S_c, S_e (Swaps for corners, edges) in Steps 4 and 5 and T_c, T_e (Twists for corners, edges) in Steps 6 and 7 were constructed using the simple moves from Steps 1 and 2. In fact, so were the moves in Step 3. In other words, the second and third layers are solved by adapting the moves used to solve the first layer. The basic idea behind the final four steps is to do swaps and twists in pairs (using the second of each pair to cancel the damage created by the first). Note that a pair of twists involves two cubies, while a pair of swaps involves three or four cubies.

Remember that preliminary rotations of the whole cube in space might be needed to put yourself into one of the cases below. In the diagrams that follow, all **CASES** are viewed from **ABOVE**.

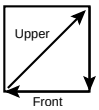
Step 4: Permute Last Layer Corners

Case I. 

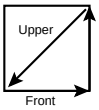
$S_c\ U^2\ S_c'\ U^2$

Case II. 

$\widehat{S}_c\ U\ \widehat{S}_c'\ U'$

Case III. 

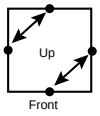
$S_c\ U\ S_c'\ U'$

Case IV. 

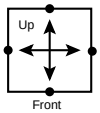
$U\ S_c\ U'\ S_c'$

Case V. None of the above. U, then one of the above cases. (U' also works.)
(This includes the common situation  .)

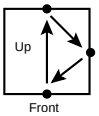
Step 5: Permute Last Layer Edges

Case I. 

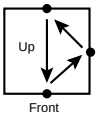
$S_e\ U^2\ S_e'\ U^2$

Case II. 

$\widehat{S}_e\ U\ \widehat{S}_e'\ U'$

Case III. 

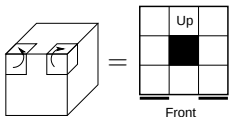
$S_e\ U\ S_e'\ U'$

Case IV. 

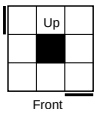
$U\ S_e\ U'\ S_e'$

Case V. None of the above. U, then one of the above cases. (U' also works.)

Step 6: Orient Last Layer Corners

Case I. 

$T_c\, U'\, T_c'\, U$

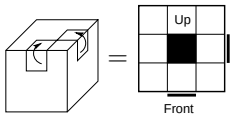
Case II. 

$T_c\, U^2\, T_c'\, U^2$

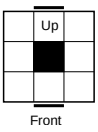
Case III. All other cases: Use repeated applications of Cases I and II. For example, for 

use Case I twice.

Step 7: Orient Last Layer Edges

Case I. 

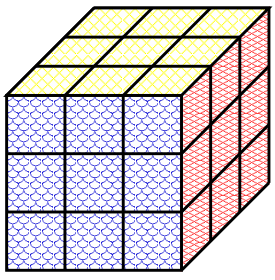
$T_e\, U\, T_e'\, U'$

Case II. 

$T_e\, U^2\, T_e'\, U^2$

Case III. All other cases: Use repeated applications of Cases I and II. Can use their inverses, too, for shorter sequences.

CONGRATULATIONS!



You have solved Rubik's Cube!