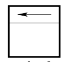
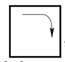
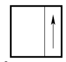
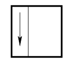
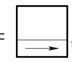
Name: ____ .

Rubik's Cube Homework 2

Notation and terminology: $U =$ , $F =$ , $R =$ , $L =$ , $D =$ , and I is the **identity move**, the move that leaves every cubie in its original position and orientation.

If X and Y are moves, then a sequence of the form $XYX'Y'$ is called a **commutator**. Note that all of the sequences given in the packet for solving the last layer are commutators. In this exercise set, we will explore why that is the case.

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1. (a) Is $UDU'D' = I$? Why or why not?

   (b) Is $RFR'F' = I$? Why or why not?

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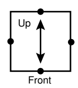
2. Suppose X does 

without affecting anything else on the Upper Layer.

   (a) What does the commutator $XUX'U'$ do to a corner cubie in the Down Layer? Explain.

   (b) What does the commutator $XUX'U'$ do to an edge cubie in the Middle Layer? Explain.

   (c) What does the commutator $XUX'U'$ do to the Upper Layer? (Give a diagram.)

   (d) What does the commutator $XU^2X'(U')^2$ do?

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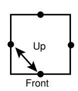
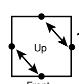
3. Suppose Y does 

without affecting anything else on the Upper Layer.

   (a) What does the commutator $YUY'U'$ do?

   (b) What commutator does 

?

If X is a move, the smallest positive integer n so that $X^n = I$ is called the **order** of X. That is, the order of X is the least number of applications of X required to return all moved cubies to their original positions and orientations. The following exercises explore the idea of order of a move.

4. (a) Find the order of L.

   (b) Find the order of U^2.

   (c) Find the order of F^3.

5. Suppose Z moves six corner cubies. One of these corner cubies is back to its original position and orientation after 3 applications of Z (and no fewer), while the other five require 15 applications of Z to return to their original positions and orientations.

   (a) What is the least number of applications of Z required to return all of the moved corner cubies to their original positions and orientations?

   (b) Suppose Z also moves seven edge cubies. All of these edge cubies require 7 applications of Z to return to their original positions and orientations. Find the order of Z, the least number of applications of Z required to move all edge and corner cubies back to their original positions and orientations.