MATH 11008 | Explorations in Modern Mathematics | Fall 2014

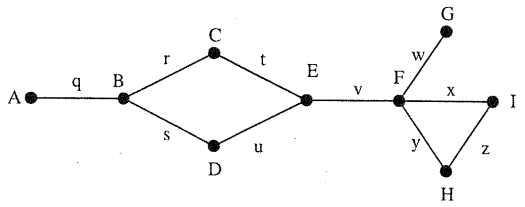
Circle one: 9:55 / 12:05 | Dr. Kracht

Name: KEY Score: /100

Exam 1 Version A

Monday, September 15, 2014

1. (20 pts) Consider the graph pictured.



(a) How many components does the graph have?

1

(b) How many vertices does the graph have?

9

(c) How many edges does the graph have?

10

(d) What is the degree of vertex B?

3

(e) What is the degree of vertex F?

4

(f) List all vertices adjacent to vertex B.

A, C, D

(g) List all edges adjacent to edge t.

r, u, v

(h) List all bridges in the graph.

q, v, w

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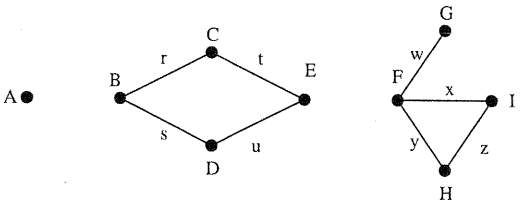
2. (10 pts) Consider the graph pictured.



(a) How many components does the graph have?

3

(b) How many vertices does the graph have?

9

(c) How many edges does the graph have?

8

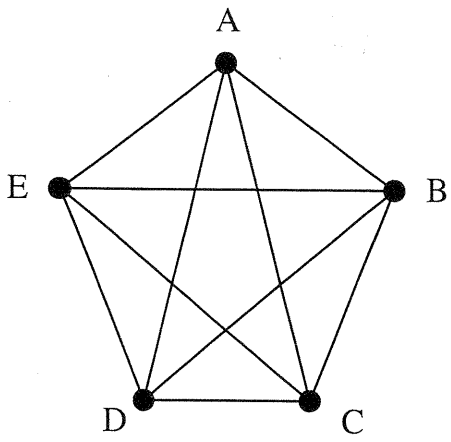
(d) What is the degree of vertex A?

0

(e) What is the degree of vertex C?

2

3. (15 pts) Consider the graph pictured to the right. *List all paths and circuits as a sequence of vertices.*



(a) Find an Euler Circuit beginning at vertex A or explain why none exists.

ABCDEACEBDA.

for example

(b) Find a Hamilton Circuit beginning at vertex D or explain why none exists.

DEABCD

for example

(c) Find a Hamilton Path from vertex C to vertex A or explain why none exists.

CDEBA

for example

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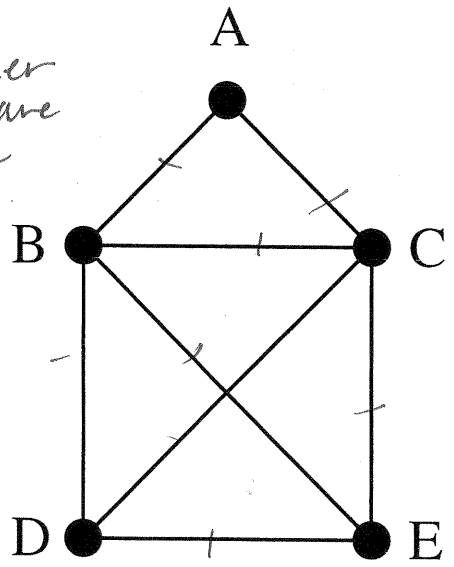
4. (15 pts) Consider the graph pictured to the right. *List all paths and circuits as a sequence of vertices.*



(a) Find an Euler Circuit beginning at vertex E or explain why none exists.

None exists. A graph has an Euler circuit if and only if all vertices have even degree. But D and E have degree 3.

(b) Find an Euler Path beginning at vertex E or explain why none exists.

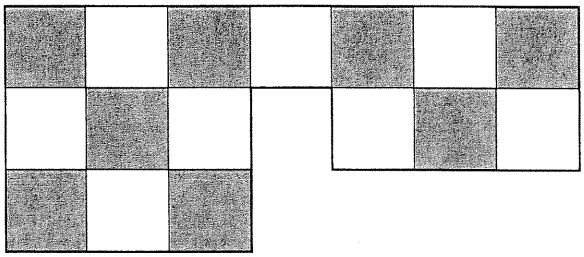
EDBACEBCD

for example

(c) Find an Euler Path beginning at vertex A or explain why none exists.

None exists. An Euler path must start on a vertex of odd degree, but A has degree 2.

6. (20 pts) Consider a Pennies and Paperclips Game with Board 3, shown.



(a) **Conjecture A:** *If Penny plays both pennies on the same color squares, Penny is guaranteed to win.*

i. Circle one: (TRUE) or FALSE

ii. Proof or counter example:

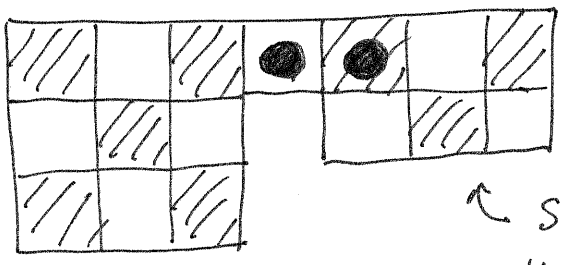
There are 8 black and 8 white squares. If Penny plays on two squares of the same color, there will be 6 of that color and 8 of the other color left. Each paperclip occupies one black and one white square. This means no more than 6 of the paperclips can be placed. Hence, Penny wins.

(b) **Conjecture B:** *If Penny plays the pennies on different color squares, Paperclip is guaranteed to win.*

i. Circle one: TRUE or (FALSE)

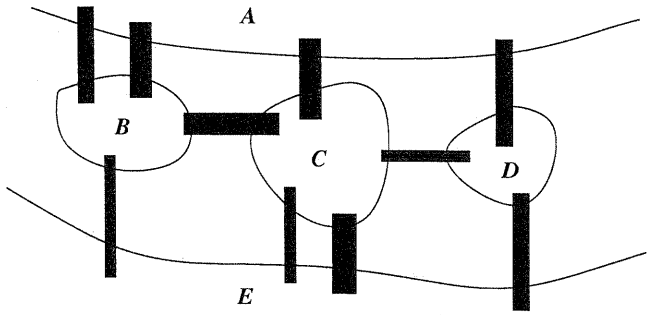
ii. Proof or counter example:

One counter example (there are many):

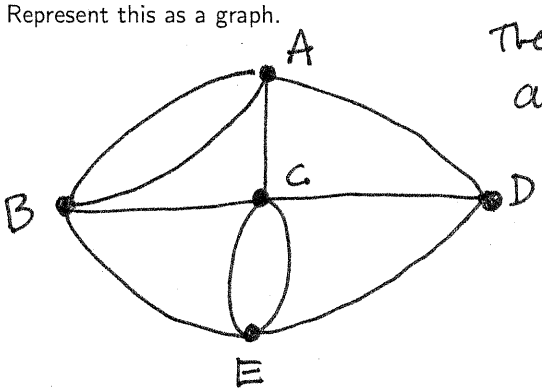


↖ since there are 5 squares on the right-hand portion of the board and the pennies are blocking the bridge to the rest of the board and since each paperclip occupies two squares, Penny wins here.

5. (20 pts) The city of Bridgeburgh has ten bridges (shaded) and land masses labeled A, B, C, D, E, as pictured.



(a) Represent this as a graph.



The vertices are land masses and edges are bridges.

degrees:

A	4
B	4
C	5
D	3
E	4

} exactly 2 odd vertices

(b) Can the residents of Bridgeburgh perambulate the town, crossing each bridge exactly once? If so, give a possible route. If not, explain why not.

Yes: for example:

DABACBECED.

(Note that all Euler paths must begin at D or C and end at the other.)

(c) Can the residents of Bridgeburgh perambulate the town, beginning and ending at the same place, and crossing each bridge exactly once? If so, give a possible route. If not, explain why not.

No. This graph has two odd vertices. Euler's Theorem says that there is an Euler Circuit in a graph if and only if all vertices in the graph have even degree.