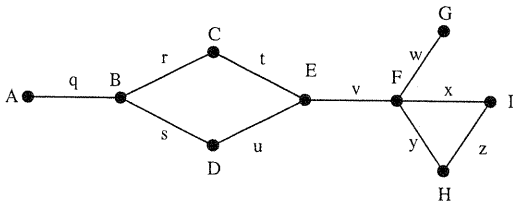


Exam 1 Version B
Monday, September 15, 2014

1. (20 pts) Consider the graph pictured.



(a) How many vertices does the graph have?

9

(b) How many edges does the graph have?

10

(c) How many components does the graph have?

1

(d) What is the degree of vertex F ?

4

(e) What is the degree of vertex E ?

3

(f) List all bridges in the graph.

q, v, w

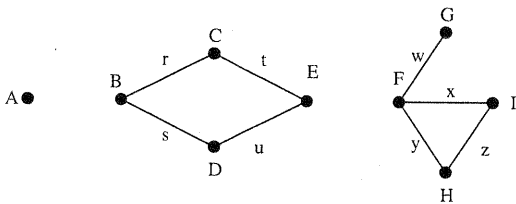
(g) List all vertices adjacent to vertex C .

B, E

(h) List all edges adjacent to edge s .

q, r, u

2. (10 pts) Consider the graph pictured.



(a) How many vertices does the graph have?

9

(b) How many edges does the graph have?

8

(c) How many components does the graph have?

3

(d) What is the degree of vertex F ?

3

(e) What is the degree of vertex A ?

0

3. (15 pts) Consider the graph pictured to the right. List all paths and circuits as a sequence of vertices.

- (a) Find an Euler Circuit beginning at vertex B or explain why none exists.

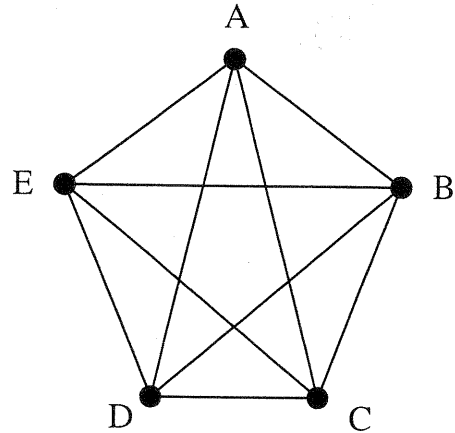
$BCDEABDACEB$
for example

- (b) Find a Hamilton Circuit beginning at vertex C or explain why none exists.

$CDEABC$

- (c) Find a Hamilton Path from vertex D to vertex B or explain why none exists.

$DEACB$



4. (15 pts) Consider the graph pictured to the right. List all paths and circuits as a sequence of vertices.

- (a) Find an Euler Circuit beginning at vertex D or explain why none exists.

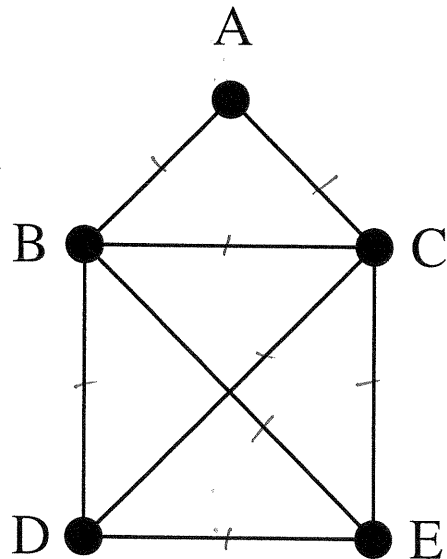
None exists. A graph has an Euler circuit if and only if all ~~the~~ vertices are of even degree. But D and E have degree 3.

- (b) Find an Euler Path beginning at vertex D or explain why none exists.

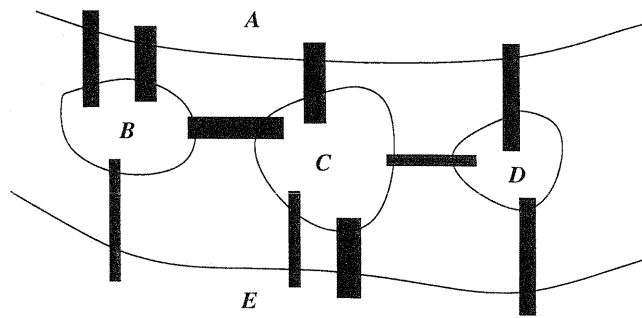
$DBACE DCBE$
for example

- (c) Find an Euler Path beginning at vertex C or explain why none exists.

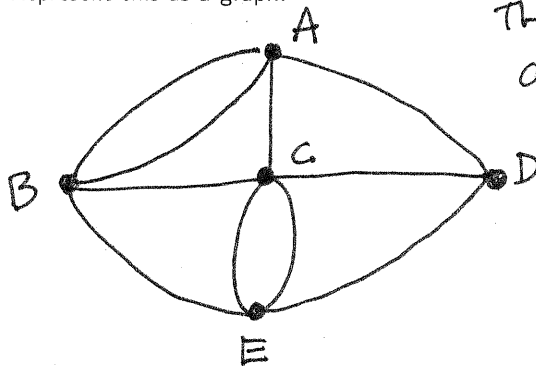
None exists. All Euler paths must begin at an odd vertex, but C has degree 4.



5. (20 pts) The city of Bridgeburgh has ten bridges (shaded) and land masses labeled A, B, C, D, E , as pictured.



- (a) Represent this as a graph.



The vertices are land masses and edges are bridges.

degrees:

A	4
B	4
C	5
D	3
E	4

} exactly 2 odd vertices

- (b) Can the residents of Bridgeburgh perambulate the town, crossing each bridge exactly once? If so, give a possible route. If not, explain why not.

Yes: for example:

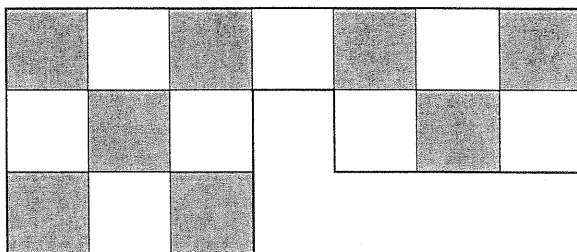
D A B A C B E C E D C .

(Note that all Euler paths must begin at D or C and end at the other.)

- (c) Can the residents of Bridgeburgh perambulate the town, beginning and ending at the same place, and crossing each bridge exactly once? If so, give a possible route. If not, explain why not.

No. This graph has two odd vertices. Euler's Theorem says that there is an Euler circuit in a graph if and only if all vertices in the graph have even degree.

6. (20 pts) Consider a Pennies and Paperclips Game with Board 3, shown.



(a) **Conjecture A:** If Penny plays both pennies on the same color squares, Penny is guaranteed to win.

i. Circle one: TRUE or FALSE

ii. Proof or counter example:

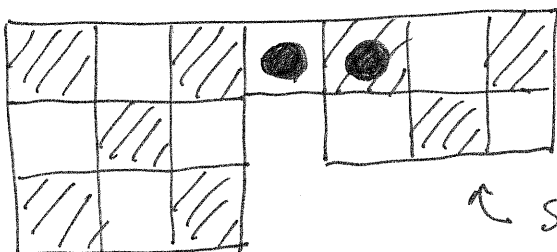
There are 8 black and 8 white squares. If Penny plays on two squares of the same color, there will be 6 of that color and 8 of the other color left. Each Paperclip occupies one black and one white square. This means no more than 6 of the paperclips can be placed. Hence, Penny wins.

(b) **Conjecture B:** If Penny plays the pennies on different color squares, Paperclip is guaranteed to win.

i. Circle one: TRUE or FALSE

ii. Proof or counter example:

One counter example (there are many):



↑ Since there are 5 squares on the right-hand portion of the board and the pennies are blocking the bridge to the rest of the board and since each paperclip occupies two squares, Penny wins here.