MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 Dr. Kracht

Print Name: _____ .

Exam 2 Version B

Friday, October 3, 2014

Academic Honesty Pledge

Your signature at the bottom indicates your agreement to abide by the following rules.

1. All purses, bags, books, notes, and other papers are placed in the designated area of the classroom.
2. Cell phones and other electronic devices (except calculators) are placed in the designated area of the classroom.
3. I will not share a calculator with another student.
4. I will not communicate with other students during the exam.
5. I will not seek help from or give help to others during the exam.
6. I will turn my exam in and will not take it from the classroom.
7. I will not discuss the exam outside of class with another student who has not yet taken the exam.
8. I will not cheat in any other way.
9. I will follow any other instructions from my professor.

Signature: _____.

Good Luck!

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Print Name: _____ Exam Score: _____/100

. *(105 pts available)*

Exam 2: Version B

F_n refers to the n^{th} Fibonacci number and $\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the golden ratio.

Part I: Long Answer.

No credit for answers without sufficient justification. Use standard mathematical notation correctly.

1. (7 pts) Write out the first fifteen terms of the Fibonacci Sequence (F_1, F_2, F_3, $\ldots$, F_{15}).

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2. (14 pts) Compute the value of each of the following. *HINT: You should be able to use your answer to the previous question for all of these.*

   (a) $F_2 + F_4 + F_5$

   (b) F_{2+4+5}

   (c) $F_2 \times F_5$

   (d) $F_{2\times 5}$

   (e) F_{F_3}

   (f) $3\,F_{10} - 3$

   (g) $3\,F_{10-3}$

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3. (4 pts) Given that $F_{32} = 2{,}178{,}309$ and $F_{33} = 3{,}524{,}578$, find F_{31}.

4. (15 pts) Recall that Φ satisfies the Golden Property, $\Phi^2 = \Phi + 1$. It is also true that $\Phi^7 = 13\Phi + 8$. Use only these facts and algebra to express Φ^8 in terms of Φ. *Show your reasoning clearly.*

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5. (15 points) Recall that T_n represents the number of tilings by pennies and paperclips of an n-board, where each penny occupies one square and each paperclip occupies two adjacent squares.

   (a) Find each of the following.

   $T_1 =$ $T_2 =$ $T_3 =$ $T_4 =$

   (b) Given that $T_{20} = 10{,}946$ and $T_{21} = 17{,}711$, find T_{22}, explaining your answer in terms of pennies, paperclips, and 22-boards.

Part II: Multiple Choice (5 points each)

Circle the letter of the best answer.

6. Kent State started with one male-female pair of baby immortal black squirrels. Immortal black squirrels begin to breed their second month. Each month, each adult pair gives birth to another male-female pair. If there were 2584 pairs in month n and 4181 pairs in month $n+1$, how many pairs of immortal black squirrels were there at month $n+2$?

   (a) 6765
   (b) 1597
   (c) 8341
   (d) 7333
   (e) None of the above

7. The golden ratio Φ is the positive solution of which of the following equations?

   (a) $x^2 = \dfrac{1}{x}$
   (b) $x = 1 + x^2$
   (c) $x^2 = 1 + x$
   (d) $x^2 = 1 - x$
   (e) None of the above

8. For large values of n, the ratio $\dfrac{F_n}{F_{n-1}}$ is approximately equal to what?

   (a) F_{n+1}
   (b) Φ^2
   (c) π
   (d) Φ
   (e) None of the above

9. Suppose that R and R' are similar rectangles. The longest side of R has length a ft and the longest side of R' has length $\frac{1}{3}a$ ft. If the perimeter of R is 30 ft, find the perimeter of R'.

   (a) 10 ft
   (b) 30 ft
   (c) 15 ft
   (d) 90 ft
   (e) None of the above

10. Suppose that R and R' are similar rectangles. The longest side of R has length a ft and the longest side of R' has length $\frac{1}{3}a$ ft. If the area of R is 36 ft^2, find the area of R'.

    (a) 108 ft^2
    (b) 324 ft^2
    (c) 4 ft^2
    (d) 12 ft^2
    (e) None of the above

11. The Lucas Numbers are defined recursively by

$L_1 = 1, \;\; L_2 = 3, \;\; L_N = L_{N-1} + L_{N-2}.$

What is L_7?

(a) 7

(b) 29

(c) 31

(d) 18

(e) None of the above

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12. The circular ring G has an inner radius of 10 and an outer radius of 16. Figure G is a gnomon to which of the following?

(a) A circular disk of radius 10.

(b) A circular disk of radius 16.

(c) A circular ring with inner radius 16 and outer radius 20.

(d) A circular ring with inner radius 10 and outer radius 32.

(e) None of the above

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13. If A is a **golden rectangle**, then which of the following is a gnomon to A?

(a) Another golden rectangle whose longer side equals the shorter side of A.

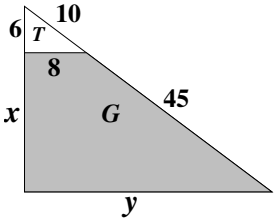
(b) Another golden rectangle whose shorter side equals the longer side of A.

(c) A square of sides equal to the longer side of A.

(d) A square of sides equal to the shorter side of A.

(e) None of the above

14. Find the value of y so that the shaded figure G is a gnomon to the white triangle T.



(a) 44

(b) 55

(c) $\dfrac{225}{4}$

(d) $\dfrac{45}{8}$

(e) None of the above

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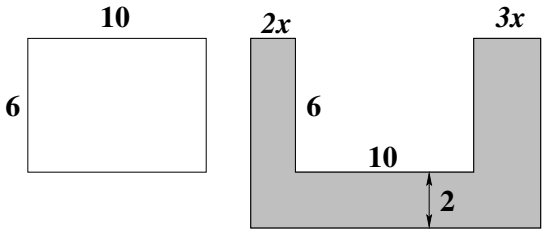
15. Find the value of x so that the shaded U-shaped region is a gnomon to the white rectangle.



(a) $\dfrac{7}{3}$

(b) $\dfrac{3}{5}$

(c) $\dfrac{5}{9}$

(d) $\dfrac{2}{3}$

(e) None of the above

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