MATH 11008 | Explorations in Modern Mathematics | Fall 2014

Circle one: 9:55 / 12:05 Dr. Kracht

Print Name: KEY Exam Score: /100

(105 pts available)

Exam 2: Version B

F_n refers to the n^{th} Fibonacci number and $\Phi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the golden ratio.

Part I: Long Answer.

No credit for answers without sufficient justification. Use standard mathematical notation correctly.

1. (7 pts) Write out the first fifteen terms of the Fibonacci Sequence ($F_1, F_2, F_3, \ldots, F_{15}$).

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610

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2. (14 pts) Compute the value of each of the following. *HINT: You should be able to use your answer to the previous question for all of these.*

(a) $F_2 + F_4 + F_5$ $= 1 + 3 + 5 = 9$

(b) F_{2+4+5} $= F_{11} = 89$

(c) $F_2 \times F_5$ $= (1)(5) = 5$

(d) $F_{2\times 5}$ $= F_{10} = 55$

(e) F_{F_3} $= F_2 = 1$

(f) $3\,F_{10} - 3$ $= 3(55) - 3$ $= 165 - 3 = 162$

(g) $3\,F_{10-3}$ $= 3F_7 = 3(13) = 39$

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3. (4 pts) Given that $F_{32} = 2{,}178{,}309$ and $F_{33} = 3{,}524{,}578$, find F_{31}.

$F_{33} = F_{31} + F_{32}$

So $F_{31} = F_{33} - F_{32}$

$= 3{,}524{,}578 - 2{,}178{,}309$

$= 1{,}346{,}269$

$$\begin{array}{r} 3\,524\,578 \\ -\;2\,178\,309 \\ \hline 1\,346\,269 \end{array}$$

Version B

4. (15 pts) Recall that Φ satisfies the Golden Property, $\Phi^2 = \Phi + 1$. It is also true that $\Phi^7 = 13\Phi + 8$. Use only these facts and algebra to express Φ^8 in terms of Φ. *Show your reasoning clearly.*

Given: $\Phi^7 = 13\Phi + 8$

$\Phi \cdot \Phi^7 = \Phi(13\Phi + 8)$

$\Phi^8 = 13\Phi^2 + 8\Phi$

$\Phi^8 = 13(\Phi + 1) + 8\Phi$ ($\Phi^2 = \Phi + 1$, Golden Property)

$\Phi^8 = 13\Phi + 13 + 8\Phi$

$\boxed{\Phi^8 = 21\Phi + 13}$

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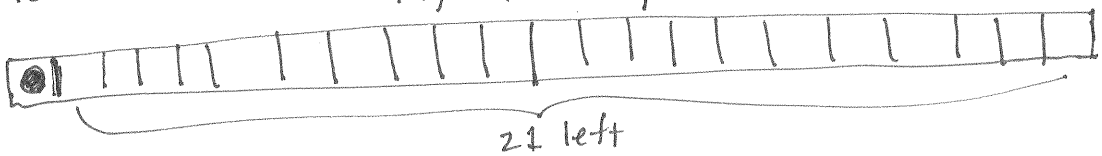
5. (15 points) Recall that T_n represents the number of tilings by pennies and paperclips of an n-board, where each penny occupies one square and each paperclip occupies two adjacent squares.

(a) Find each of the following.

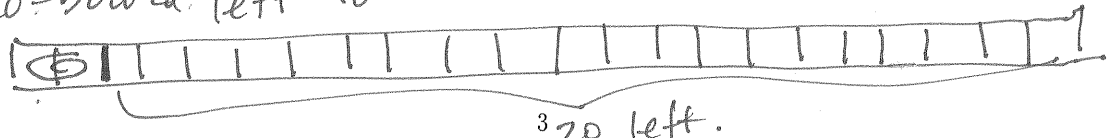
$T_1 = 1$ $\quad T_2 = 2$ $\quad T_3 = 1+2 = 3$ $\quad T_4 = 2+3 = 5$

(b) Given that $T_{20} = 10{,}946$ and $T_{21} = 17{,}711$, find T_{22}, explaining your answer in terms of pennies, paperclips, and 22-boards.

Every tiling of the 22-board must start with either a penny or a paperclip. If it starts with a penny, then it leaves a 21-board to tile. There are 17,711 ways to do that.



If it starts with a paperclip, then there is a 20-board left to tile. There are 10,946 ways to do that.



Hence, there are a total of $T_{21} + T_{20} = 17{,}711 + 10{,}946 = 28{,}657$ ways to tile a 22-board. ∎

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Part II: Multiple Choice (5 points each)

Circle the letter of the best answer.

6. Kent State started with one male-female pair of baby immortal black squirrels. Immortal black squirrels begin to breed their second month. Each month, each adult pair gives birth to another male-female pair. If there were 2584 pairs in month n and 4181 pairs in month $n+1$, how many pairs of immortal black squirrels were there at month $n+2$?

A

(a) 6765
(b) 1597
(c) 8341
(d) 7333
(e) None of the above

2584
4181
6765

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7. The golden ratio Φ is the positive solution of which of the following equations?

C

(a) $x^2 = \frac{1}{x}$
(b) $x = 1 + x^2$
(c) $x^2 = 1 + x$
(d) $x^2 = 1 - x$
(e) None of the above

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8. For large values of n, the ratio $\frac{F_n}{F_{n-1}}$ is approximately equal to what?

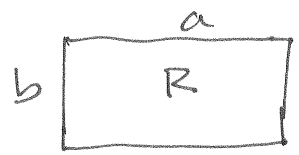
D

(a) F_{n+1}
(b) Φ^2
(c) π
(d) Φ
(e) None of the above

9. Suppose that R and R' are similar rectangles. The longest side of R has length a ft and the longest side of R' has length $\frac{1}{3}a$ ft. If the perimeter of R is 30 ft, find the perimeter of R'.

A

(a) 10 ft
(b) 30 ft
(c) 15 ft
(d) 90 ft
(e) None of the above



$P = 2a + 2b$
$= 30$ ft

$P' = 2(\frac{1}{3}a) + 2(\frac{1}{3}b)$
$= \frac{1}{3}(2a + 2b)$
$= \frac{1}{3}(30) = 10$ ft

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10. Suppose that R and R' are similar rectangles. The longest side of R has length a ft and the longest side of R' has length $\frac{1}{3}a$ ft. If the area of R is 36 ft^2, find the area of R'.

C

(a) 108 ft^2
(b) 324 ft^2
(c) 4 ft^2
(d) 12 ft^2
(e) None of the above

See figure above:

$A = ab = 36$ ft^2

$A' = (\frac{1}{3}a)(\frac{1}{3}b)$
$= \frac{1}{9}(ab)$
$= \frac{1}{9}(36)$
$= 4$ ft^2

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11. The Lucas Numbers are defined recursively by

$L_1 = 1,\ L_2 = 3,\ L_N = L_{N-1} + L_{N-2}.$

What is L_7?

B

(a) 7
(b) 29
(c) 31
(d) 18
(e) None of the above

$L_1 = 1$
$L_2 = 3$
$L_3 = 4$
$L_4 = 7$
$L_5 = 11$
$L_6 = 18$
$L_7 = 29$

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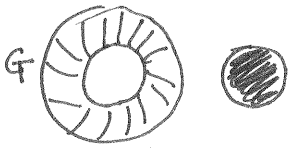
12. The circular ring G has an inner radius of 10 and an outer radius of 16. Figure G is a gnomon to which of the following?

A

(a) A circular disk of radius 10.
(b) A circular disk of radius 16.
(c) A circular ring with inner radius 16 and outer radius 20.
(d) A circular ring with inner radius 10 and outer radius 32.
(e) None of the above



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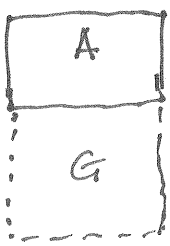
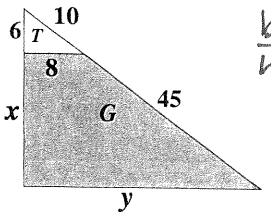
13. If A is a **golden rectangle**, then which of the following is a gnomon to A?

C

(a) Another golden rectangle whose longer side equals the shorter side of A.
(b) Another golden rectangle whose shorter side equals the longer side of A.
(c) A square of sides equal to the longer side of A.
(d) A square of sides equal to the shorter side of A.
(e) None of the above



14. Find the value of y so that the shaded figure G is a gnomon to the white triangle T.



A

(a) 44
(b) 55
(c) $\frac{225}{4}$
(d) $\frac{45}{8}$
(e) None of the above

base : hypotenuse

$\frac{y}{10+45} = \frac{8}{10}$

$y = \frac{8 \cdot 55}{10}$

$y = \frac{2 \cdot 4 \cdot 5 \cdot 11}{2 \cdot 5}$

$y = 4 \cdot 11$
$= 44$

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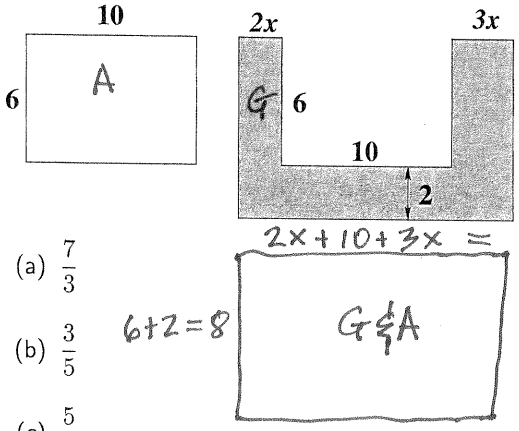
15. Find the value of x so that the shaded U-shaped region is a gnomon to the white rectangle.



D

(a) $\frac{7}{3}$
(b) $\frac{3}{5}$
(c) $\frac{5}{9}$
(d) $\frac{2}{3}$
(e) None of the above

long : short

$\frac{5x+10}{8} = \frac{10}{6}$

$\frac{5x+10}{8} = \frac{5}{3}$

$24 \cdot \left(\frac{5x+10}{8}\right) = \frac{5}{3} \cdot 24$

$3(5x+10) = 5 \cdot 8$
$15x + 30 = 40$
$15x = 10$
$x = \frac{10}{15} = \frac{2}{3}$